Contributions to Modelling Correlations in Financial Econometrics

Ayesha Scott
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Abstract

The importance of modelling the correlations between the returns of financial assets has long been recognised in the field of portfolio management. In his 2003 Nobel Prize lecture, Robert Engle identified two new frontiers for future research in the field of volatility modelling: large dimensional multivariate models and high frequency volatility models. The aim of this thesis is to contribute to these two ongoing areas of the correlation modelling literature. In the context of large dimensional problems, the thesis presents a practical empirical framework to assess a number of models used to generate correlation forecasts for the purposes of portfolio allocation. Evidence is found in favour of assuming equicorrelation across various portfolio sizes, in particular during times of market turbulence. The equicorrelation framework is then extended to allow the correlation structure to be conditional on volatility, leading to superior portfolio allocation outcomes. Further, the benefit of assuming equicorrelation is found to be limited when forecasting correlations between indices, rather than equities. The findings documented here provide useful insights into the best way to handle large dimensional problems and the behaviour of models designed to forecast the correlations of such systems. In terms of intraday data sampled at high frequencies, very little work exists on the dynamics of correlations during the trading day despite research into modelling intraday volatilities gaining momentum. This thesis outlines important features of intraday correlation dynamics and proposes a novel multivariate GARCH approach to model these processes. Models that capture both an intraday pattern and daily persistence of the correlations provide promising results over the sample. These findings further the understanding of intraday volatilities and correlations, a topic relevant to many financial applications.

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

QUT Verified Signature

Ayesha Scott
Date: 12 July 2016

Acknowledgments

> *"...burning with curiosity, she ran across the field after it, and fortunately was just in time to see it pop down a large rabbit-hole under the hedge.*
>
> *In another moment down went Alice after it, never once considering how in the world she was to get out again."*
>
> from Alice's Adventures in Wonderland (Lewis Carroll 1865, p. 12)

The PhD is assumed to be a solitary pursuit; something completed in isolation. My experience has been the opposite and so there are many people to thank for their time and energy as I dove headfirst down my own rabbit-hole.

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Commonly Used Abbreviations

- AR: Autoregressive
- ARCH: Autoregressive Conditional Heteroscedasticity
- CCC: Constant Conditional Correlation
- cDCC: Consistent DCC
- cDCC-CL: Consistent DCC (Composite Likelihood)
- DCC: Dynamic Conditional Correlation
- DCC-ARE: Volatility Dependent DCC (Additive; Regime)
- DCC-AVE: Volatility Dependent DCC (Additive; Level)
- DCC-TVR: Volatility Dependent DCC (Time-varying; Regime)
- DCC-TVV: Volatility Dependent DCC (Time-varying; Level)
- DEC-ARE: Volatility Dependent DECO (Additive; Regime)
- DEC-AVE: Volatility Dependent DECO (Additive; Level)
- DECO: Dynamic Equicorrelation
- DEC-TVR: Volatility Dependent DECO (Time-varying; Regime)
- DEC-TVV: Volatility Dependent DECO (Time-varying; Level)
- EQ-W: Equally Weighted
- EWMA: Exponentially Weighted Moving Average
- GARCH: Generalised Autoregressive Conditional Heteroscedasticity
- GFC: Global Financial Crisis
- GMV: Global Minimum Variance
- IC: Information Criterion
- MCS: Model Confidence Set
- MGARCH: Multivariate GARCH
- MIDAS: MIxed DAta Sampling
- MPT: Modern Portfolio Theory
- MS: Markov Switching
- SACF: Sample Autocorrelation Function

- SMA: Simple Moving Average
- VDCC: Volatility Dependent DCC
- VDECO: Volatility Dependent DECO
- VIX: Volatility Index

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Chapter 1

Introduction

1.1 Overview

"The ability to define what may happen in the future and to choose among alternatives lies at the heart of contemporary societies."

Peter L. Bernstein (1996, p. 2)

At the heart of financial decision making is the concept of risk. The ability to forecast the risk inherent in financial asset returns, often linked to the variability or volatility of the returns, is a foundation of security markets and informs an investor's optimal behaviour. Investor decisions occurring today must be based on some expectation of what will occur in the future. Expectations of the volatility of security returns are a key input into decision making processes. One thing complicating these decisions is that a typical portfolio is made up of many securities, each one risky, potentially interacting with the others. Thus, not only does an expectation of volatility in the returns of each individual asset need to be determined, but an expectation of the correlation between them is required to capture the risk of the entire portfolio.

In his 2003 Nobel Prize lecture, Robert Engle identified two new frontiers for future research in the field of volatility modelling: large dimensional multivariate models and high frequency volatility models. The former deals with generating a risk profile for portfolios containing a vast number of assets and the latter seeks to exploit the information contained in data sampled at short time intervals over a trading day. This thesis contributes to both strands of literature, providing insights into the optimal way to deal with these issues as the search continues for superior methods to those used currently. Questions will be addressed concerning large dimensional correlation matrices and the models available to forecast them. This thesis also contributes to the burgeoning literature examining the complications that arise when modelling the correlation matrix at high frequencies. From a practical point of view, as institutions face greater pressure to manage risks effectively (EY, 2014) and with increased reliance on automated trading, good intraday correlation forecasts are crucial.

1.2 Key Concepts

Prior to any specific discussion of the key research questions posed in this dissertation, some terminology needs to be defined. All empirical work begins with a dataset of *asset returns*. Generated from the history of prices of a financial security, returns discussed in this document are specifically *log returns*, computed as $[\log(p_{n,t}) - \log(p_{n,t-1})]$ where $p_{n,t}$ is the price of asset n at time t. Second is the idea of *forecasting*, defined here as the analysis of past information in order to predict future correlations and/or volatility. These forecasts are generated for the purpose of informing portfolio management decisions. In the context of financial econometrics, *volatility* is defined as the dispersion of asset returns. Whichever way it is measured it is inherently unobservable. Modelling of such a process inevitably involves use of a proxy for the volatility and for the purposes of much of this thesis, the square of the asset returns series is used. Lastly, the idea of what constitutes a *large* portfolio, or *large dimensional* problem requires definition. Practically, a large portfolio would be one that contains hundreds, if not thousands, of assets. As will become clear throughout the chapters that follow, the optimal method for coping with these systems remains an open question. Subsequently, for the purposes of empirical work the somewhat vague *hundreds* is refined to a number of assets greater than 50.[1]

Figure 1.1 shows the daily returns of the S&P 500 stock market index, r_t, the top panel, and the corresponding squared returns, r_t^2. The sample period spans from 3 January 1996 through to 31 December 2014. The plot clearly displays common characteristics of the

[1]See Engle and Sheppard (2001) for discussion of portfolio size in the context of multivariate volatility modelling.

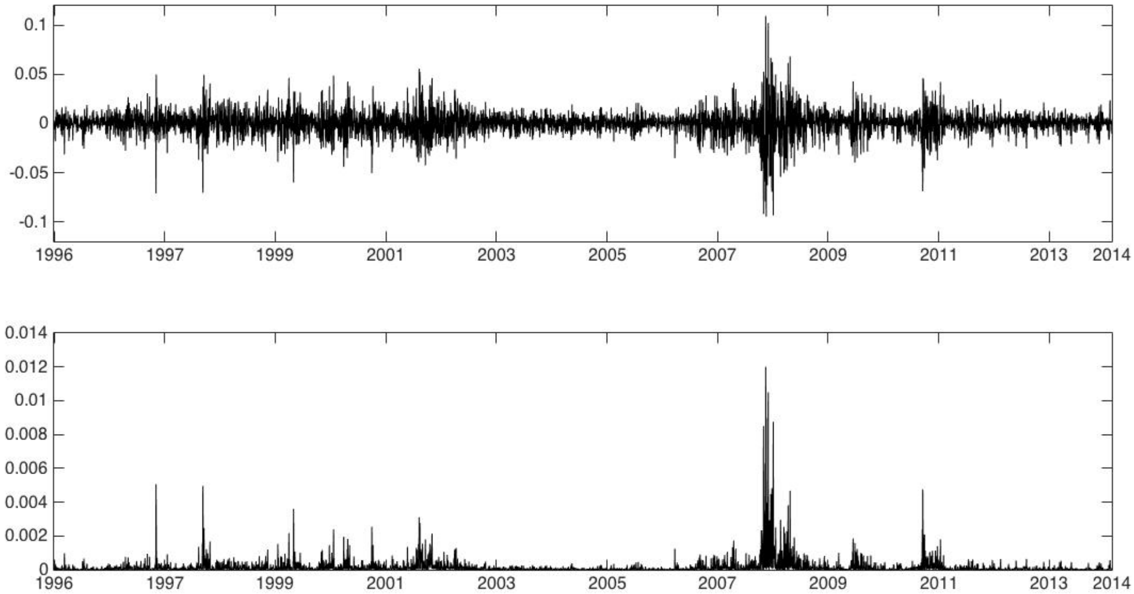

Figure 1.1: Daily returns, r_t, of the S&P 500 index (top) and squared daily returns of the index, r_t^2 (bottom). Period spans 3 January 1996 to 31 December 2014.

volatility process of financial securities. One of the most important is the clustering evident in squared returns, where a large move is followed by another large move irrespective of the direction. The reverse, where a small move is followed by a small move, is also true. This persistence implies that the expectation of tomorrow's volatility depends on that observed today. The idea that volatility is predictable has inspired a vast body of research framed around estimating and forecasting these processes accurately. Despite this extensive work, the requirement for a multivariate model that can handle estimating volatilities and correlations for a large number of assets, such as the portfolio of a large mutual fund, remains a practical and relevant problem.

The returns in Figure 1.1 are sampled at a daily frequency, however trading occurs throughout the day and thus there is an opportunity to sample price information almost continuously. The ability to collect and use high frequency intraday data for the purposes of volatility and correlation modelling has become possible due to advances in computing power. Researchers are now focusing on ways to exploit the information contained in intraday data, ranging from using high frequency data to generate lower frequency (daily) measures of volatility through to modelling the intraday volatility process itself. An open question is how best to model intraday correlation dynamics, as sampling price information at high frequency gives rise to various complications not seen at lower frequencies.

1.3 Key Research Questions

The overarching theme of this thesis is the modelling of correlations for a portfolio of financial assets. The three research questions outlined here provide a comprehensive look at various aspects of correlation modelling of asset returns, including the large dimensional context and high frequency intraday correlation dynamics.

1. Are complex specifications for modelling correlations necessary or do simpler moving average based methods produce adequate forecasts of large dimensional correlation matrices?

There are numerous models aimed at forecasting volatility and correlation, however the ability to generate adequate forecasts in the large dimensional setting remains elusive. Models from the MGARCH family are compared to semi-parametric, moving average style models that are much easier to implement. This comparison is important given a number of problems such as parameter proliferation and computational burden[2] that arise in higher dimensions. A portfolio allocation application is used to evaluate the performance of the estimators, across a range of portfolio sizes and subsamples of high and low volatility.

2. Can the equicorrelation framework be improved by exploiting the link between volatility and correlations?

In answering the previous research question, evidence of potential benefits of assuming equicorrelation in large dimensions is found. Additionally, the differing performance of models between subsamples of comparatively high and low volatility indicates some relationship exists between volatility and correlations. To investigate possible improvements in equicorrelation, this link between volatility and correlations is exploited by conditioning the equicorrelation structure on market volatility. Analysis of this relationship is presented in two empirical examples, with both a national (U.S.) and international (Europe) context studied. The various correlation forecasting methods are compared using a portfolio allocation problem.

[2] *Computational burden* is used here to describe the actual runtime of the models themselves, as well as the idea that increased computer power does not address slow runtimes as dimension increases.

3. What are the features of pairwise correlations within a trading day and how can they be modelled?

The modelling of univariate intraday volatility dynamics has benefited from extensive studies of its intraday behaviour, however little has been done in the context of multivariate intraday correlations. The thesis investigates the interesting features of pairwise intraday correlations and how to account for these unique characteristics in a modelling framework. This is the first study to explicitly document such patterns with a view toward modelling these dynamics. An MGARCH approach is presented, specifically Dynamic Conditional Correlation, for estimating the intraday correlations. A corresponding set of equicorrelated models is also provided. The novel use of the MGARCH framework captures the daily persistence evident in pairwise correlations and takes into account the intraday pattern seen over the trading day. A portfolio of Australian equities is used to examine the new set of models in terms of full sample fit. The dataset is also split into sub-portfolios of differing levels of unconditional correlation to extend analysis of the intraday pattern and provide further insight into modelling these processes. Additionally, possible applications and extensions of the study are provided to highlight future research avenues for the modelling of intraday correlations.

1.4 Key Contributions

This thesis makes a number of contributions to the literature on correlation modelling. Firstly, the thesis assesses the performance of various correlation forecasting models in the context of large dimensional multivariate problems and provides insight into the superior models for this task. The practical nature of the empirical framework used to evaluate the correlation forecasts provides evidence of the economic value of the correlation forecasting methods studied. The conditions under which they may perform optimally are identified by dividing the time series into subsamples based on relative high and low volatility. Evidence in favour of the MGARCH framework is found, with an assumption of equicorrelation proving useful for a range of portfolio sizes. However, in the large portfolio case, the suitability of the Constant Conditional Correlation model during periods of market calm can not be discounted. These results confirm earlier work (Engle and Kelly, 2012 and Laurent, Rombouts and Violante, 2012) and further it by specifically considering the case of higher dimensions. The implication is that complex specifications of the correlation matrix such as MGARCH-based methods are preferred over simpler moving average style models, however in certain settings very basic versions of the MGARCH class are adequate.

Secondly, by conditioning the correlation structure on market volatility the thesis finds a volatility dependent structure in general leads to improved portfolio outcomes. Significant insights into the use of the equicorrelation and Dynamic Conditional Correlation models are gained, furthering the understanding of these models and their suitability in a range of circumstances. Comparison of the two empirical examples presented suggests contrasting findings, namely the benefit of equicorrelation is limited when forecasting correlations between indices, rather than individual equities.

Third, the thesis considers the intricacies of modelling correlations in high frequency intraday data, finding intraday characteristics of the pairwise correlation processes evident over the trading day. By studying high frequency returns of equities traded on the Australian Stock Exchange, an inverted U-shape pattern is identified in the intraday correlations between assets. Further, this intraday pattern is most evident between stocks that have a lower level of unconditional correlation, such as those from different industries.

Lastly, the thesis presents a novel use of the MGARCH approach, specifically Dynamic Conditional Correlation and Dynamic Equicorrelation, for estimating the intraday correlations. This framework allows for persistence at the daily level, evident in pairwise correlations, and also captures the intraday pattern seen over the trading day. It is found modelling both persistence at a daily frequency and the intraday diurnality is a promising avenue for future work in this area.

1.5 Thesis Structure

Chapter 2 provides a comprehensive motivation of volatility timing, outlining the history of time series modelling that is relevant to this thesis. Definitions of important concepts are provided and background given on the diverse range of approaches developed in this literature. The aim of this chapter is to provide a detailed overview of the body of literature to motivate and position this thesis within the field of correlation, and more widely volatility, forecasting.

Chapter 3, titled *'On the Benefits of Equicorrelation for Portfolio Allocation'*[3] investigates a number of correlation forecasting models, specifically considering their usefulness in the context of large portfolios. This chapter seeks to answer the first research question and uses a portfolio allocation exercise to compare computationally simple moving average based methods to the relatively complex MGARCH correlation forecasting methods. Chapter 4, titled *'Volatility Dependent Dynamic Equicorrelation'*[4] provides a link between the standard equicorrelation framework and volatility, addressing the second research question. The motivation for this chapter is partly due to the benefits of assuming equicorrelation found in Chapter 3, as the apparent advantages of this framework in the portfolio allocation context is worth further examination. Chapter 5, titled *'Modelling Intraday Correlations using Multivariate GARCH'* shifts the focus from methods directly forecasting the correlations of large multivariate systems to the volatility of returns sampled at high frequency and correspondingly modelling intraday correlation dynamics over the trading day. Lastly, Chapter 6 provides a summary of the key contributions of this thesis, reiterating the research questions asked in Section 1.3 and the conclusions drawn throughout the empirical work. A brief outline of where research regarding the forecasting of large correlation matrices, equicorrelation and modelling of intraday correlation may proceed in the future is also provided.

[3]A paper of the same name has been published from the research contained in this chapter, co-authored with Adam Clements and Annastiina Silvennoinen. The published version appears in the *Journal of Forecasting* (2015), Volume 34, Issue 6, pp. 507–522.

[4]Comments received from the NZESG 2015 meeting in Brisbane, especially Robert Reed, are gratefully acknowledged.

Chapter 2

Literature Review

2.1 Introduction

Recognition of the importance of volatility in finance, specifically in terms of portfolio allocation and risk management, dates back to the seminal works of Markowitz (1952) and Merton (1971). This chapter provides an overview of literature on modelling volatility and correlations relevant to empirical work contained in this thesis. Important concepts are defined and explained, along with a history of Modern Portfolio Theory and the correlation forecasting literature, motivating the use of the particular forecasting techniques seen later.

Some discussion of scope is necessary as the size of the literature is too big to accommodate here. Topics such as stochastic volatility[1] and option pricing[2] are beyond the scope of this thesis. Given the focus of the empirical work outlined in Chapter 1, the volatility and correlation forecasting methods discussed in this review are based on simplistic smoothing techniques such as moving averages through to the more complex Generalised Autoregressive Conditional Heteroscedasticity (GARCH) family of models. The comparative ease with which GARCH-type forecasting can be undertaken speaks to the practical empirical applications contained in the thesis. In GARCH modelling the conditional variance is extracted given past information with quasi-maximum likelihood used for inference.

[1] Despite the similar objective of volatility modelling, stochastic volatility is a distinct class of model. See Shephard and Andersen (2009) and Chib, Omori and Asai (2009) for reviews of this literature.

[2] Black and Scholes (1972) and Merton (1973) proposed a model to evaluate the pricing of options and it remains widely used today, with extensive research directed at forming volatility forecasts for this purpose.

For a portfolio of financial assets, such as those formed in Chapters 3 to 5, multivariate systems can be modelled using a decomposition of the covariance matrix popularised by Engle (2002). This allows the individual volatility process of each asset to be estimated and then used to scale the returns series to enable modelling of the pairwise correlation dynamics. The common two stage estimation procedure of multivariate GARCH (MGARCH) systems lends the literature review an obvious structure. First, a discussion of methods concerning the volatility process of individual assets, the so-called *first stage*, is provided. The review then progresses to a thorough examination of correlation modelling, the *second stage*.

To put the forecasting methods into context Section 2.2 begins with an overview of the volatility timing literature, motivating the large and varied field of volatility modelling. This includes a discussion of mean-variance portfolio theory, provided as the basis for the empirical applications. Preliminary discussion of volatility and correlation, in terms of how each is defined and measured, and their respective characteristics is contained in Section 2.3. The Volatility Index (VIX) is relevant to empirical work contained in Chapter 4, accordingly it is defined in Section 2.3.3. Methods of forecasting the volatility of univariate time series are outlined in Section 2.4, in line with the scope of this review described above. The univariate volatility discussion forms the basis for much of the multivariate work contained in Section 2.5. The multivariate models are integral to the research contained in this thesis and a thorough examination of these methods is provided. Section 2.6 considers the ways in which volatility forecasts are evaluated, including the use of loss functions, the Model Confidence Set and economic value. Section 2.7 concludes.

2.2 The Importance of Volatility Timing

The dynamic nature of volatility and the fact that its level varies over time has long been acknowledged empirically. Researchers such as Schwert (1989), among others, have found that the average level of volatility in equity returns is higher during economic recessions than during periods of economic growth. The quest to explain and predict changes in volatility dynamics has led researchers to apply a wide variety of approaches with mixed success. This search for an optimal forecasting model that can accurately capture the characteristics of volatility and aid investor decision making is motivated by the evolution of Modern Portfolio Theory. Given that volatility plays such an integral role in finance it stands to reason that substantial effort has been devoted to this problem. This section aims to motivate the key questions posed by this thesis, providing relevant background that proves to be key knowledge built upon by the rest of the literature explored in this chapter.

2.2.1 Modern Portfolio Theory

A natural starting point of any discussion of portfolio allocation and subsequently the importance of volatility forecasting to investor decision making, is the mean-variance theory of Markowitz (1952). The significance of volatility timing in a portfolio allocation sense however, was documented by Merton (1971).[3]

The basis of Modern Portfolio Theory is the Markowitz model, whereby an investor's decisions are based solely on expected return and risk. It is assumed that risk averse investors act to maximise their one-period utility,[4] where their utility curves demonstrate diminishing marginal utility of wealth. The expected return of a portfolio of N assets is the weighted average of the expected return of each asset n, $\mathbb{E}(r_n)$,

$$\mathbb{E}(r_{port}) = \sum_{n=1}^{N} w_n \mathbb{E}(r_n) = \boldsymbol{w}'\mathbb{E}(\boldsymbol{r}) \ , \tag{2.1}$$

where $\boldsymbol{w}$ is a $N \times 1$ vector of weights and $\boldsymbol{r}$ a $N \times 1$ vector of asset returns. Risk in the Markowitz context is measured as the variance (or standard deviation) of the expected returns,

$$\sigma^2_{port} = \sum_{n=1}^{N}(w_n^2\sigma_n^2) + \sum_{n=1}^{N}\sum_{m=1}^{N}(w_n w_m Cov_{n,m}) = \boldsymbol{w}'\mathbf{H}\boldsymbol{w} \ , \qquad n \neq m \ . \tag{2.2}$$

The risk of a portfolio is minimised for a given estimate of the covariance matrix, $\mathbf{H}$ by an adjustment of the security weights, $\boldsymbol{w}$. The covariance matrix, $\mathbf{H}$ has variances along

[3] Similarly, Tobin (1958) associated the risk in a portfolio with the variance of its returns. See Elton and Gruber (1997) for a detailed review of Modern Portfolio Theory.

[4] Several researchers have examined extending the single-period problem to a multi-period setting (see Mossin, 1968, Fama, 1970, Hakansson, 1974 and Merton, 1990) and concluded that under a set of reasonable assumptions the multi-period problem can be solved simply as a sequence of single period problems.

the diagonal and covariances elsewhere, therefore

$$\mathbf{H} = \begin{bmatrix} \sigma_1^2 & Cov_{12} & \cdots & Cov_{1N} \\ Cov_{21} & \sigma_2^2 & \cdots & Cov_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Cov_{N1} & Cov_{N2} & \cdots & \sigma_N^2 \end{bmatrix} \tag{2.3}$$

for the individual assets making up a given portfolio. Each security exhibits covariation with the others in the portfolio, shown here as the sample estimator $Cov_{n,m}$

$$Cov_{n,m} = \frac{1}{T-1} \sum_{t=1}^{T} (r_{n,t} - \bar{r}_n)(r_{m,t} - \bar{r}_m) \ , \tag{2.4}$$

where $\bar{r}_n$ and $\bar{r}_m$ denote the sample means of the respective returns series. The optimal portfolio is the portfolio that offers the highest expected return for a given level of risk.[5]

2.2.2 Global Minimum Variance Portfolio

Practically, the estimation of $\mathbb{E}(r_n)$ is difficult as the sensitivity of security weights to small changes in forecasted returns causes problems when evaluating the volatility forecasts. The global minimum variance (GMV) portfolio simply minimises risk with no input from a returns perspective. This unique characteristic means the GMV portfolio is of particular interest in all empirical portfolio allocation applications in this thesis. A number of studies involving minimum variance portfolios have been conducted, motivated by this convenient fact. For example, Clarke, de Silva and Thorley (2006) confirm the earlier work of Haugen and Baker (1991) that minimum variance portfolios can be shown to add value above a strategy based on market capitalisation weighted portfolios. The GMV portfolio with optimal weights $\boldsymbol{w}$ is

$$\min_{\boldsymbol{w}} \boldsymbol{w}'\mathbf{H}\, \boldsymbol{w} \ , \tag{2.5}$$

[5] Several researchers have suggested expanding the number of moments included above and beyond that of the simple mean-variance portfolio theory (see Kraus and Litzenberger, 1976, Lee, 1977, Harvey and Siddique, 2000, and Brockett and Kahane, 1992), however the Markowitz theory remains the cornerstone of Modern Portfolio Theory. Elton and Gruber (1997) suggest this is in part due to the appeal of mean-variance theory, as it is an intuitive and well-developed process of portfolio selection.

solved subject to the budget constraint $\mathbf{1}'\boldsymbol{w} = 1$. The weights are

$$\boldsymbol{w}_{GMV} = \frac{\mathbf{H}^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{H}^{-1}\mathbf{1}} , \tag{2.6}$$

where $\mathbf{1}$ is a vector of ones.

The benefits of using the GMV portfolio extend to evaluation of covariance forecasts and the application of the GMV portfolio in this context is discussed further in Section 2.6. This section and the last have highlighted the need for accurate and practical estimates of volatility and covariances to better inform investor decision making for the purposes of portfolio allocation. Indeed, Andersen, Bollerslev, Christoffersen and Diebold (2005, p. 13) highlight the importance of covariance forecasting as *"...at least as important as volatility forecasting in the context of financial asset pricing..."*. The following sections further define the concepts of volatility and covariance in the context of financial assets and outline their characteristics, focusing on those seen in equity returns.

2.3 Volatility and Correlation

Consider here the return of an asset n as

$$r_n = \sigma_n \, \epsilon_n \qquad n = 1,\ 2,\ \dots,\ N , \tag{2.7}$$

where σ_n is the standard deviation and ϵ_n a disturbance term. The return in equation 2.7 can be thought of as a series of returns collected over some time period T, with individual time steps denoted t, in

$$r_{n,t} = \sigma_n \, \epsilon_{n,t} \qquad n = 1,\ 2,\ \dots,\ N . \tag{2.8}$$

The returns series in equation 2.8 infers that the volatility of asset n, that is the standard deviation σ_n, is unconditional. In the next section, several *stylised facts* of volatility are identified, questioning the assumption that σ_n is time invariant. This outline of the common features of volatility frames the discussion of how volatility is defined and measured, including an explanation of what is considered to be important when determining volatility proxies. In terms of the key characteristics of volatility, the scope here is specifically equity returns. Although many of the characteristics and stylised facts hold for other markets such as bonds and futures, it is equities (and stock market indices) that are the focus of this thesis. Also relevant to the empirical applications is the Volatility Index, referred to as the VIX, and details as to its construction are contained in Section 2.3.3. Finally, Sections 2.3.4 and 2.3.5 expand on the univariate case by considering the comovement of a pair of assets n and m, extending the review to the multivariate context.

2.3.1 Characteristics of Volatility

"The general consensus is that financial asset return volatilities have a predictable component which is dependent on the past volatilities and return shocks."

Tsui and Yu (1999, p. 503)

The seminal work of Merton (1971) was among the first papers to formally introduce the concept of time varying volatility and suggest that this characteristic or stylised fact of volatility would be useful in the portfolio allocation context. Fleming, Kirby and Ostdiek (2001) show that there is a significant economic gain in timing volatility for portfolio allocation, motivated by empirical studies that show (among other things) the time varying nature of volatility. The returns series described in equation 2.8 above is now extended to be

$$r_{n,t} = \sigma_{n,t}\, \epsilon_{n,t} \qquad n = 1,\ 2,\ \dots,\ N\ , \tag{2.9}$$

where $\sigma_{n,t}$ is the conditional standard deviation. In addition to being dynamic over time, most researchers also consider volatility to be mean reverting (see for example Poon and Granger, 2003). That is, there is some level of volatility that will be returned to given a long enough time scale. Although generally considered a characteristic of volatility processes, there is some disagreement in the literature regarding what this level is and how resistant to change it is given certain factors. As a result, modelling univariate volatilities in terms of both short run and long run components has featured in recent literatures, see Engle, Ghysels and Sohn (2013) for an example. A useful illustration of the ongoing discussion of the concept of mean reversion is given in Section 2.3.3, which outlines a benchmark for U.S. stock market volatility, and discusses regime changes in the level of volatility over time.

The fact that volatility exhibits persistence is arguably the most important characteristic of the variance of asset returns, first recognised by Mandelbrot (1963), and later by Fama (1965), Chou (1988) and Schwert (1989). That is, the clustering of volatility shocks so that a large move (of either sign) will be followed by another large move and so forth. It is this persistence to which Tsui and Yu (1999) refer in the remark quoted above. Persistence implies that the expectation of volatility in the future depends on shocks observed today. Under this assumption, the conditional first and second moments of the returns process $r_{n,t}$ are directly observable and gives rise to the use of squared returns $r^2_{n,t} = \hat{\sigma}^2_{n,t}$, a proxy of the volatility discussed further in Section 2.3.2. It also forms the basis of a raft of popular methods for forecasting volatility and correlations, more on which is discussed in Sections 2.4 and 2.5.

Another characteristic of the volatility process of equity returns is the so-called *leverage effect* or asymmetry in the volatility. That is, the impact of negative news is larger than that of positive news. One of the oldest stylised facts of volatility, this feature was introduced in the work of Black (1976) and later Christie (1982) and Schwert (1989). Asymmetry has been exploited for modelling purposes by numerous researchers including Nelson (1991), Zakoïan (1994) and Glosten, Jagannathan and Runkle (1993), and these models are discussed in detail in Section 2.4. An extensive literature related to the impact of news on volatility is available, beginning with work such as Pagan and Schwert (1990), Campbell and Hentschel (1992) and Engle and Ng (1993),[6] all of whom note asymmetry empirically.

In addition to asymmetry, equity returns also display leptokurtosis or heavy tails. Put simply, they are fat-tailed, containing a higher number of large events than that described by the standard Normal distribution. Empirically, this characteristic has been discussed by Mandelbrot (1963) and Fama (1965) among others. Numerous authors have subsequently suggested use of suitably fat-tailed distributions, such as the Student t distribution. Examples of early work in this area include Clark (1973) and Blattberg and Gonedes (1974).

Finally, it is also evident that the decline of shocks in asset returns is not exponential but that they decay at a slower rate. This is often termed *long memory*. The degree

[6] Of particular note is Engle and Ng (1993), with the notion of the 'news impact curve', used to measure how new information is accounted for in the volatility process.

of integration of these series has prompted researchers to consider fractionally integrated processes when describing these dynamics, see Baillie, Bollerslev and Mikkelsen (1996). The fractionally integrated models, discussed further in Section 2.4.2, allow explicit relationships with past information and thus lend themselves to forecasting in keeping with other popular methods of modelling.

A good forecasting model will allow for some of these characteristics, however not all features of the volatility process need to be captured for a model to be effective empirically. Of additional importance to the quality of forecasts is the measure used to estimate the volatility, as well as the proxy used to evaluate the estimate and assess its quality.

2.3.2 Defining and Measuring Volatility

In the context of financial econometrics, volatility is defined as an analysis of risk, or a measure of dispersion of asset returns. It is usually measured using the standard deviation of these returns, as in equation 2.9. Following on from this, variance (statistically) is standard deviation squared, σ_t^2, although both standard deviation and variance are often termed volatility. By whatever measure is used, it is an inherently latent process. Modelling of such a process inevitably involves use of some proxy, $\hat{\sigma}_t^2$, for the volatility and subsequently the addition of error into the experiment. Andersen and Bollerslev (1998) discussed the issue of adequate predictions, suggesting that it is the noise inherent in the unbiased squared return innovation, $r_t^2 = \hat{\sigma}_t^2$, that leads to judging volatility models to have low predictive power. They argue that standard volatility models do produce reasonable forecasts and propose using high frequency intraday data to form a more accurate measure of volatility. Hansen and Lunde (2006) furthered this argument and show that *realized volatility*, shown here as

$$RV_t = \hat{\sigma}_t^2 = \sum_{i=1}^{I} r_{t,i}^2 \ , \tag{2.10}$$

allows for better ranking of models out-of-sample. The number of intraday observations at time t is represented by I and i is an index of those observations.

A large amount of discussion around the topic of using high frequency data has been concerned with the optimal sampling frequency used to form the realized volatility (RV).

The idea of RV is not a new one, with Merton (1980) documenting that a perfect measure of volatility can be estimated given continuous data of an asset price. However, this is not the case in practice, where the price process is observed at discrete intervals. It has also been shown that there are market microstructure effects that need to be considered, as these impact on the accuracy of the RV estimator. These microstructure effects mean the recorded prices do not reflect a true diffusion process, for example the difference between the buy and sell price of an asset (the bid-ask spread). Aït-Sahalia, Mykland and Zhang (2011) classify a market microstructure effect as belonging to one of three groups. The first includes those that occur naturally during trading such as the bid-ask spread and prices differing between markets. Secondly, they define informational effects, for example the gradual price response to a large trade and inventory control. The third group is one of errors in the measurement of prices, including prices entered incorrectly or as zero. They show that data collected on a tick-by-tick basis to form the RV will in fact be a measure of the variance of the market microstructure noise rather than the volatility of the price process under study. As a result, rather than including all available data the sampling frequency is often taken at a larger interval, commonly between 5 to 30 minutes (see Andersen, Bollerslev, Diebold and Ebens, 2001, Andersen, Bollerslev and Meddahi, 2011 and Ghysels and Sinko, 2011, among others).

Other volatility proxies, in addition to squared returns and RV, have been suggested in the literature. The corrected intraday range of Patton (2011) is one such alternative, shown as

$$RG_t^{*2} = \hat{\sigma}_t^2 = \left[\frac{\max_\tau \log P_\tau - \min_\tau \log P_\tau}{2\sqrt{\log(2)}} \right]^2 , \qquad t-1 < \tau \leq t \, , \tag{2.11}$$

where $2\sqrt{\log(2)}$ corrects for bias in the unadjusted intraday range. Patton (2011) asserts that each of the abovementioned proxies will be unbiased, assuming a Brownian motion process for the log returns. Despite the use of a proxy, estimation leading to reasonable predictions of the volatility process can be undertaken and these forecasts are useful for a range of financial applications.

2.3.3 The Volatility Index (VIX)

This section details another measure of volatility, offering a more general take on the concept by providing an idea of market volatility. It is specifically relevant to Chapter 4 and so described here. Often referred to as the *investor fear gauge*, the Chicago Board Options Exchange (CBOE) implied Volatility Index (VIX) is considered the benchmark for U.S. stock market volatility. It is designed to measure the 30-day expectation of volatility of the S&P 500 Index and is calculated with out-of-the-money put and call options that have between 23 and 37 days to expiration, over a range of strike prices. See CBOE (2014) for technical details regarding calculation and Whaley (2000) for a general overview of its premise. By construction it is model-free, that is, no underlying option pricing model is used. Originally introduced in 1993, the methodology underpinning the VIX was updated in 2003.[7] It has been a tradable asset in the form of VIX futures and options since 2004 and 2006 respectively.

Figure 2.1 shows the VIX and daily returns of the S&P 500 market index, emphasising the relationship between the index and the VIX. Over the sample period, the VIX reached its highest point of 80.86 in November 2008 during the Global Financial Crisis (GFC). Whaley (2009) emphasised the *forward looking* nature of the VIX and Szado (2009) outlined the diversification benefits of long positions in VIX securities during the GFC.

Whaley (2009), Fleming, Ostdiek and Whaley (1995) and Giot (2005b), among others, found an asymmetric and negative relationship between returns and implied volatility measured by an index such as the VIX. In a similar vein, the relationship between the VIX and news sentiment was considered by Smales (2014) who found evidence of a significant asymmetric negative relationship. That is, negative news results in larger changes in the VIX than positive news. Fleming (1998), Becker, Clements and White (2006) and Becker, Clements and McClelland (2009) also studied the informational content of implied volatility, examining it from the view of market efficiency.

A second body of literature investigates methods to model the VIX itself. Structural breaks and regime shifts have been identified by Guo and Wohar (2006), Baba and Sakurai (2011) and Sarwar (2012). Each study found three regimes of pre-1992, 1992-1997

[7]The new methodology was then used to generate historical prices for the VIX, going back to 1990. The original Volatility Index (VXO) was based on S&P 100 Index options.

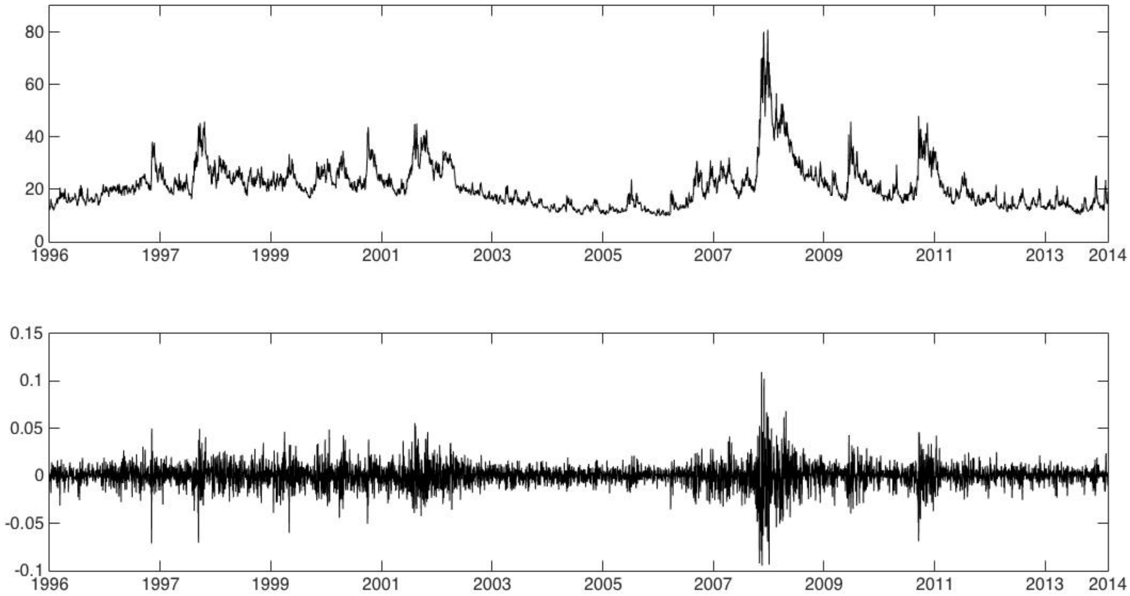

Figure 2.1: The Volatility Index, or VIX (top) and daily returns of the S&P 500 index (bottom). Period spanning 3 January 1996 to 31 December 2014.

and post-1997. More recently, Sarwar (2014) found two distinct regimes, over a sample period beginning in 1998 and ending 2013. This sample is similar to the period shown in Figure 2.1. Baba and Sakurai (2011) used a Markov switching model to study macroeconomic variables as leading indicators of implied volatility regime shifts and identified term spreads as influencing a change in VIX regime. Further discussion of regime switching models, in particular Markov switching, is contained in Section 2.4.3.

2.3.4 Defining and Measuring Correlation

Attention now turns to the volatility of a portfolio of assets: the multivariate case. Before describing the characteristics of these processes and expanding on the complications of modelling them, it is useful here to provide some definition of the comovement between a pair of assets n and m. Such relative covariation is termed *covariance*, discussed in terms of mean-variance theory and portfolio allocation in Sections 2.2.1 and 2.2.2. Correlation[8] is a comparable measure of this comovement. It is defined as the covariance divided by

[8] Specifically the sample 'Pearson correlation coefficient'.

the product of the standard deviations of each of the assets. In the two asset case it is

$$\rho_{n,m} = \frac{1}{T-1} \sum_{t=1}^{T} \left(\frac{r_{n,t} - \bar{r}_n}{\sigma_n} \right) \left(\frac{r_{m,t} - \bar{r}_m}{\sigma_m} \right) , \tag{2.12}$$

where T is the the number of observations in the sample; $\bar{r}_n$ and $\bar{r}_m$ are the means of the returns series', $r_{n,t}$ and $r_{m,t}$, for assets n and m over the sample; and, σ_n and σ_m are the standard deviations of each asset.

In the univariate context, volatility or σ_t, is latent and a proxy, $\hat{\sigma}_t$, used. This is also true of the multivariate correlation process. Again, a proxy must be identified. The outer product of returns is used to generate the multivariate covariance proxy $\widehat{\mathbf{\Sigma}}_t$

$$\widehat{\mathbf{\Sigma}}_t = \boldsymbol{r}_t \boldsymbol{r}_t' \,. \tag{2.13}$$

Additionally, the so-called *realized covariance* is the outer product of the realized volatilities

$$RCOV_t = \widehat{\mathbf{\Sigma}}_t = \sum_{i=1}^{I} \boldsymbol{r}_{t,i} \boldsymbol{r}_{t,i}' \,. \tag{2.14}$$

Bauwens, Braione and Storti (2014) defined realized covariance as an estimate of the volatility of the returns matrix based on higher frequency returns. In this thesis, their definition is narrowed to be the covariance at a daily frequency, given intraday returns taken at intervals throughout the trading day (for example every 5 minutes). For discussion of more precise evaluation of volatility forecasts in the multivariate case see Becker, Clements, Doolan and Hurn (2015).

In addition to correlation, the term *equicorrelation* is important to the empirical work contained in later chapters. Equicorrelation is defined as equal pairwise correlations between all assets in a portfolio at a particular point in time.[9] It is often identified as ρ with the equicorrelated matrix, $\mathbf{R}$,

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix} . \tag{2.15}$$

[9]Early work on the statistical analysis of equicorrelated vector samples dates back to Basu (1972).

For the purposes of this thesis, this definition is extended to the equicorrelation being dynamic through time and its value is denoted ρ_t. The time varying nature of covariance and correlation is discussed further below.

An effort has been made in the empirical chapters of the thesis to distinguish between the ideas of the volatility of an individual asset and the comovement or correlation between a pair of such assets. In general however, the literature groups these terms together under the umbrella of *volatility*.

2.3.5 Characteristics of Correlation

The covariation between pairs of assets in a portfolio share common characteristics that can inform modelling their dynamics, and subsequently forecasting. This discussion also leads to a preview of the complications that arise in the multivariate context.

Foremost, the structure of the correlation matrix is important. It must have ones on the diagonal (as each asset is perfectly correlated with itself) and each correlation coefficient is subject to the boundary constraint $-1 < \rho < 1$. As is the case for covariance matrices, the correlation matrix needs to be symmetric and positive definite. This is an important feature of these matrices, and one that poses potential problems for researchers developing correlation models. Section 2.5 goes into detail about models appropriate for the multivariate context and how this issue has been approached in the literature.

Given the basic foundations of what constitutes a covariance or correlation matrix, common features evident in the comovements between pairs of financial asset returns series' (specifically equity returns) can be highlighted. Firstly, as an extension of the dynamic nature of univariate volatility, covariance (and correlation) is time varying. Recognition of its temporal dependence goes back a long way, see for example Bollerslev, Engle and Wooldridge (1988) for an asset pricing example. In addition, it is generally considered to be mean reverting, although as in the one asset case this is subject to some controversy. For example, recent work by Bauwens et al. (2014) proposed a conditional covariance model that includes a dynamic long-run component in addition to a mean reverting short-run process. This type of framework allows the covariances to mean revert over a short horizon but accounts for occasional events (such as crises) that cause shifts in correlation dynamics over the longer term.

Secondly, covariances appear to exhibit long memory. Like the univariate case, shocks decline slowly. Models accounting for long memory are a continuing area of interest in the literature. Recent examples include Níguez and Rubia (2006), who forecast the conditional covariance matrix of a portfolio of assets that exhibit long memory, and the panel model of Luciani and Veredas (2015).

Lastly, the correlations between equity returns series display asymmetry. That is, correlations seem to respond to negative news more strongly than positive news of the same magnitude. Cappiello, Engle and Sheppard (2006) provide reasoning for this behaviour, asserting that a negative shock puts downward pressure on a pair of stocks and increases their respective variances. Under the assumptions of mean-variance theory, covariance will increase if the respective risk of each stock in relation to the market doesn't change. If the individual variances do not change proportionally then the correlations will also increase, see equation 2.12. Several researchers have suggested multivariate models accounting for this asymmetry, including the *news impact surface* of Kroner and Ng (1998), a multivariate extension of the *news impact curve* (Engle and Ng, 1993).

Developing a model that successfully estimates (and thus forecasts) all the empirical facts listed here, in both the univariate and multivariate cases, is the ultimate goal of those concerned with volatility and correlation forecasting. In the absence of the perfect model, evaluation of the forecasting methods becomes important and this issue is discussed in Section 2.6.

2.4 Univariate Time Series Forecasting

This section reviews the ways in which the volatility of individual financial assets have been modelled. Emphasis is placed on the methods relevant to the empirical work is this thesis. Discussion begins with what Engle (2004) termed *historical volatility*, where the volatility is estimated over some window and that is then assumed to be the volatility for the next period. Assigning equal weights to these past observations is perhaps an unrealistic scheme, and Section 2.4.1 outlines extensions to the basic premise of these methods. The generalised autoregressive conditional heteroscedasticity (GARCH) models pioneered by Engle (1982) and Bollerslev (1986) are highlighted in Section 2.4.2. Section 2.4.3 focuses on regime switching models, already touched on in Section 2.3.3. Lastly, Sections 2.4.4 and 2.4.5 review univariate models using high frequency intraday returns, both in the context of realized volatility (the RV in equation 2.10) and intraday volatility.

2.4.1 Historical Volatility

The persistent nature of volatility implies that future volatility is dependent on the past, discussed in Section 2.3.1. This characteristic of the volatility of financial asset returns motivates the use of moving averages and smoothing techniques to form volatility predictions. Each is practical and quick to compute, advantages that make moving averages a popular tool of technical traders and investors.

The simplest forecasting model to generate the variance h^2 at time t is a simple moving average

$$h_t^2 = \frac{1}{K} \sum_{k=1}^{K} r_{t-k}^2 \ , \tag{2.16}$$

where K is the moving average period (referred to as the *rolling window*) and r_{t-k}^2 the kth lag of historical squared returns (the estimate of the lagged variance of the series).

The basic moving average in equation 2.16 can be extended to the exponentially weighted moving average,

$$h_t^2 = \alpha \, h_{t-1}^2 + (1 - \alpha) \, r_{t-1}^2 \ . \tag{2.17}$$

This model places a higher emphasis on more recent observations. Once again, h_t^2 is the forecast of volatility and r_{t-1}^2 the lagged squared returns. The parameter α is constrained to lie between 0 and 1 and is commonly referred to as the rate of decay. J.P. Morgan/Reuters (1996) RiskMetrics examined the exponential filter in detail, and provided optimal decay rates for a range of scenarios and data. Examples of appropriate decay rates applicable to data used in this thesis include the U.S. equity market returns, $\alpha = 0.98$, and general daily equity data $\alpha = 0.94$.

The MIxed DAta Sampling (MIDAS) regression of Ghysels, Santa-Clara and Valkanov (2006) is motivated by a desire to have a flexible, parsimonious specification to estimate future volatility based on data that may be sampled at a different frequency. It is shown as

$$h^2_{t+j} = \mu_j + \phi_j \sum_{k=0}^{K} b_j(k,\theta)\tilde{X}_{t-k,t-k-1} + \epsilon_{j,t} \tag{2.18}$$

where h^2_{t+j} is the j-step ahead forecast of volatility; μ_j the unconditional (mean) volatility; scale parameter ϕ_j; K is the truncation point of the k lags for the regressor $\tilde{X}$; and, the polynomial lag parameters, or weights, $b_j(k,\theta)$.

The weights are not unrestricted parameters but rather a function of θ, allowing for a longer memory specification (a characteristic of volatility described in Section 2.3.1). The way in which θ is specified is dependent on the problem at hand, however the Beta function is commonly used in keeping with Ghysels et al. (2006).[10] The resulting weights are normalised to sum to one, allowing estimation of the scale parameter ϕ_j.

The MIDAS framework is versatile enough to support any regressor $\tilde{X}$, or set of regressors, suitable to forecast future volatility. These regressors can be sampled at a different frequency to the volatility forecast h^2_{t+j}. MIDAS is also easily adaptable to give multi-period forecasts, emphasised in equation 2.18 with the $t+j$, j-step forecast. Although Ghysels et al. (2006) was primarily concerned with predicting return volatility using various types of regressors including squared returns, absolute returns and realized volatility (among others), the literature utilising MIDAS regressions contains a range of applications.

This section has highlighted popular smoothing techniques built on the premise of volatility persistence, namely the idea that predictions of future movements in asset returns are dependent on the past. From the basic simple moving average through to the flexible MIDAS regression, the methods contained in this section are computationally simple in nature and thus have gained traction in practice. The following section introduces a different approach to the same idea of persistence, providing an overview of the empirically successful generalised autoregressive conditional heteroscedasticity (GARCH) family of models.

[10] Other weighting schemes have been explored in the literature, including an exponentially weighted specification in Engle et al. (2013). Ghysels and Valkanov (2012) also show weighting schemes or polynomial lag parametrisations such as Almon, Exponential Almon and linear step function are possible.

2.4.2 The (G)ARCH Universe

The concept of volatility persistence in financial asset returns coupled with the specification of $r_{n,t}$ in equation 2.9 is the basis of the empirically successful ARCH family of model, one of the most notable advancements in volatility modelling. First proposed by Engle (1982), the ARCH model allows the conditional variance, h_t, to vary over time, dependent on the past squared forecast errors. It is defined

$$h_t = \omega + \sum_{q=1}^{Q} \alpha_q r_{t-q}^2 \qquad q = 1,\ 2,\ \dots,\ Q\ , \tag{2.19}$$

where the parameters are constrained as $\omega > 0$, $\alpha_q \geq 0$ and $\sum_{q=1}^{Q} \alpha_q < 1$. The seminal work of Engle has since become the basis of an increasingly large number of generalisations.

The most commonly applied extension of ARCH is the Generalised ARCH (GARCH) model of Bollerslev (1986), a successful predictor of conditional variances even in its simplest form. The GARCH (P, Q) model is mean reverting and conditionally heteroscedastic with a constant unconditional variance. It is defined as

$$h_t = \omega + \sum_{q=1}^{Q} \alpha_q r_{t-q}^2 + \sum_{p=1}^{P} \beta_p h_{t-p} \tag{2.20}$$

where h_t is the univariate variance at time t and h_{t-p} the pth lag; r_{t-q}^2 the squared return at time $t-q$; and ω, α_q and β_p parameters constrained to $\omega > 0$, $\alpha_q \geq 0$, $\beta_p \geq 0$ and $\sum_{q=1}^{Q} \alpha_q + \sum_{p=1}^{P} \beta_p < 1$. The weights on the squared returns decline geometrically at a rate estimated from the data. In essence this generalises the ARCH model into an autoregressive moving average, allowing for a more flexible lag structure than its predecessor.

Many variations on the ARCH model are available, including ARCH-M, IGARCH and TARCH to name only a few.[11] Another of the particularly influential models is the Exponential GARCH (EGARCH) of Nelson (1991), which recognises the asymmetric nature of volatility for equity returns. Volatility reacts asymmetrically to past forecast errors such that in a financial sense, negative returns seem to have a larger influence on future volatility than positive ones, as discussed in Section 2.3.1. The EGARCH model addressed this characteristic of volatility by incorporating the sign of a return, rather than

[11] A recent survey of Hansen and Lunde (2005) compared the out-of-sample performance of 330 ARCH-type models.

its magnitude alone. It is shown as

$$\log(h_t^2) = \omega + \sum_{q=1}^{Q} \alpha_q(|z_{t-q}| - \mathbb{E}(|z_{t-q}|)) + \sum_{q=1}^{Q} \phi_q z_{t-q} + \sum_{p=1}^{P} \beta_p \log(h_{t-p}^2) \tag{2.21}$$

where $z_t = h_t^{-1} r_t$ and asymmetry is captured if the parameter $\phi_q > 0$. The logarithms avoid non-negativity constraints on the parameters and guarantee the estimated variance will be positive.

Similarly, the model of Glosten et al. (1993) (GJR-GARCH) addresses asymmetry in volatility by including a dummy variable that takes the value 1 should the asset return be negative. The specification of such a model is

$$h_t = \omega + \sum_{q=1}^{Q}(\alpha_q + \phi_q D_{t-q}) r_{t-q}^2 + \sum_{p=1}^{P} \beta_p h_{t-p} \tag{2.22}$$

where D_{t-q} is the indicator variable at time $t - q$ and ϕ_q the relevant parameter. The constraints of the original model in equation 2.20 become $\omega > 0$, $\alpha_q + (\phi_q/2) \geq 0$, $\beta_p \geq 0$ and $\sum_{q=1}^{Q}(\alpha_q + (\phi_q/2)) + \sum_{p=1}^{P} \beta_p < 1$. An alternative approach is to consider asymmetric distributions. For example, Harvey and Siddique (1999) develop an extended GARCH model to model skewness directly. They assume a non-central t-distribution, in contrast to the models described in equations 2.30 through 2.22 where errors are assumed to be i.i.d. and in general, normally distributed.

It is worth noting that a fitted GARCH (1,1) model often displays near integrated (IGARCH) or non-stationary behaviour. A number of explanations for this behaviour have been suggested, including long memory and structural breaks (or both). Structural breaks or changes in regime can be addressed using a regime switching model, discussed in Section 2.4.3. To account for both long memory and occasional break dynamics, researchers including Baillie and Morana (2009) and Kılıç (2011) have suggested methods based on the Fractionally Integrated GARCH (FIGARCH) model.

Briefly mentioned in Section 2.3.1, the FIGARCH model of Baillie et al. (1996) is designed to account for volatility's characteristic long memory. It addresses a criticism of the original GARCH, namely that its lagged weighting scheme fails to account for the long memory exhibited by the volatility process. Specification of the model, using the lag operator L, is

$$h_t = \omega[1-\beta(1)]^{-1} + \left\{1 - [1-\beta(1)]^{-1}\phi(L)(1-L)^d\right\}\epsilon_t^2 \tag{2.23}$$

where

$$\begin{aligned}
\phi(L) &= [1-\alpha(L)-\beta(L)](1-L)^{-1} \\
(1-L)^d &= 1 - dL + \tfrac{d(d-1)}{2!}L^2 - \tfrac{d(d-1)(d-2)}{3!}L^3 + \cdots \\
\alpha(L) &= \alpha_1 L + \cdots + \alpha_q L^q \\
\beta(L) &= \beta_1 L + \cdots + \beta_p L^p
\end{aligned} \tag{2.24}$$

For FIGARCH, the degree of integration, d, governs the rate at which volatility shocks decay. When $0 < d < 1$, the rate of decay is hyperbolic and slower than the original GARCH model in equation 2.20, hence it is commonly referred to as long memory GARCH.

There is also a relationship between long memory processes and aggregation of possibly dependent dynamic series. Granger (1980) illustrates that aggregating a number of short memory processes can result in the appearance of long memory. This has given rise to a body of work in this area, including the Heterogeneous Autoregressive (HAR) framework, discussed in Section 2.4.4 in the context of realized volatility.

The discussion of MIDAS regressions in the previous section can be linked to the GARCH family as researchers develop more sophisticated volatility forecasts. Recent work by Engle et al. (2013) introduced the GARCH–MIDAS model, a component model relating volatility to macroeconomic variables. The macroeconomic variables (for example, inflation and growth) drive a long-run component sampled monthly or quarterly and volatility observed daily. A mean-reverting GARCH accounts for short-term fluctuations. Another GARCH-based model related to the MIDAS regressions is the so-called HYBRID–GARCH of Chen, Ghysels and Wang (2010, 2011). These GARCH models aim to forecast volatility at frequencies different to that of the information set, with the name derived from High frequencY data-Based pRojectIon-Driven (HYBRID). The authors loosely define the model as a GARCH version of the MIDAS framework.

The advent of the GARCH family of models has proven empirically useful in the field of finance, providing researchers with the possibility of many interesting applications over the last decade or so. Despite the numerous variations on the most basic form of the model, Hansen and Lunde (2005) have shown that it is difficult to better the forecasts of a basic GARCH(1,1), with a leverage effect, for equity returns.

2.4.3 Regime Switching

In addition to the characteristics of volatility outlined in Section 2.3.1, there is empirical evidence of shifts in the behaviour of financial time series under certain conditions, for example economic crises. The GARCH-based models outlined above generally assume mean reversion. They therefore lack the flexibility to allow for shifts in the unconditional level of volatility, h. Processes exhibiting regime changes can be expressed using the Markov switching (MS) models. Suppose that there is some time series (asset returns, for instance)

$$y_t = \mu_{S_t} + \epsilon_t \tag{2.25}$$

where ϵ_t is normally distributed with a 0 mean and variance of $\sigma^2_{S_t}$. The state $S_t = 1, 2, ..., s$ and represents shifts in the dynamics of the time series, y_t. In a two state world, $s = 2$, equation 2.25 can be expressed as

$$\text{State 1} : y_t = \mu_1 + \epsilon_t \qquad \epsilon_t \sim (0, \sigma_1^2) \tag{2.26}$$

$$\text{State 2} : y_t = \mu_2 + \epsilon_t \qquad \epsilon_t \sim (0, \sigma_2^2)\ . \tag{2.27}$$

There are two different volatilities governing the dynamics of y_t, σ_1^2 and σ_2^2. The switching dynamics, that is how y_t transitions from State 1 to State 2, is governed by a transition matrix that contains probabilities of switching from one state to another. This is the most basic of switching models, however it is straightforward to extend the system above to handle more complex dynamics.

For example, a two state MS-autoregressive (AR) model in the context of volatility can be shown as

$$v_t = \kappa_{S_t} + \lambda_1 v_{t-1} + \lambda_2 v_{t-2} + \sigma_{\eta,S_t}\eta_t \tag{2.28}$$

where v_t is the level of volatility and, κ_{S_t} and σ_{η,S_t} are state-dependent parameters that switch according to the state $S_t \in \{0, 1\}$, driven by an unobserved Markov process. In this example, the low volatility state can be denoted by $S_t = 0$ and high volatility by $S_t = 1$.

An interesting application of regime switching models in the context of univariate volatility forecasting is analysis of the VIX (see Section 2.3.3). Sarwar (2012) tested for multiple structural breaks in the VIX, confirming the earlier work of Guo and Wohar (2006) in identifying 3 structural shifts: pre-1992, 1992–1997 and post-1997. In their study of macroeconomic variables as leading indicators of VIX regime shifts, Baba and Sakurai (2011) used a three state Markov Switching model over their time period of 1990 to 2010 and also found shifts similar to Guo and Wohar (2006). The work of Sarwar (2014), who studied a period of 1998 to 2013, identified two distinct regimes.

Allowing for regime changes is useful for a range of applications with researchers mixing existing models with Markov processes to model volatility, for example the Markov switching MIDAS specification of Guérin and Marcellino (2013). ARCH models with regime shifts have been motivated by possible structural change in the ARCH process, see Hamilton and Susmel (1994). The so-called SWARCH model is specified

$$r_t = \sqrt{g_{S_t}} \times \tilde{r}_t \ , \tag{2.29}$$

where $\tilde{r}_t = \sqrt{h_t}\, \epsilon_t$ and ϵ_t is a disturbance term, with

$$h_t = \omega + \sum_{q=1}^{Q} \alpha_q \tilde{r}_{t-q}^2 \qquad q = 1,\ 2,\ \dots\ ,\ Q\ . \tag{2.30}$$

The underlying variable $\tilde{r}_t$ is multiplied by the constant $\sqrt{g_{S_t}}$, where S_t is the regime state and governed by an unobserved Markov chain. The SWARCH model of Hamilton and Susmel (1994) provided better forecasts than standard ARCH models, which are thought to exhibit too much persistence when faced with a shock of high magnitude (such as a stock market crash).

This section has described only a few of the regime switching models, outlining those relevant to the empirical work of this thesis and also the key developments in this area. Similar techniques to those highlighted here have been extended to multivariate systems, elaborated on in Section 2.5.3.

2.4.4 Realized Volatility

Previous sections of this review have assumed squared daily returns as the proxy for latent volatility. Recently, significant improvements have been made in the way researchers measure volatility. The most notable of these has been *realized volatility* (RV) which uses high frequency data collected throughout the trading day (intraday data) to form the volatility at a lower frequency. It has been explored by authors such as Andersen and Bollerslev (1998) and Hansen and Lunde (2006), among others. Recall from Section 2.3.2,

$$RV_t = \hat{\sigma}_t^2 = \sum_{i=1}^{I} r_{t,i}^2 \ , \tag{2.31}$$

where the number of intraday observations on day t is represented by I and i is an index of those observations. For a comprehensive review of the realized volatility literature, see Andersen, Bollerslev, Diebold and Labys (2003) and more recently McAleer and Medeiros (2008).

The MIDAS approach described in Section 2.4.1 readily lends itself to applications using realized volatility, as do extensions of the FIGARCH specification of Baillie et al. (1996) in Section 2.4.2. Both allow for the long memory characteristic of the volatility process. The general form of the ARFIMA(P,d,Q) process is also popular in this context,

$$[1 - \beta(L)](1 - L)^d RV_t = \omega + [1 + \alpha(L)]u_t \tag{2.32}$$

where $\alpha(L)$ and $\beta(L)$ are coefficient polynomials of order P and Q and u_t is an innovation. The degree of fractional integration is d. If $d = 0$, the ARFIMA becomes an ARMA(P,Q) process. To ensure positive variances and that the effect of the lagged observations reasonably describes the volatility process, the constraint $0 \leq d \leq 1$ is imposed. The ARFIMA framework is used in numerous applications, see Baillie (1996) and Andersen, Bollerslev, Diebold and Labys (2001, 2003) among others.

Taking a different approach is the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV), see Corsi (2009). The HAR-RV is a simple linear regression that uses RV_t over heterogeneous intervals as regressors (it can be thought of as similar to the MIDAS regression framework in Section 2.4.1). The HAR framework is in essence a good approximation to long memory models and specified as

$$RV_{t+1,t} = \mu + \beta^D RV_t^D + \beta^W RV_t^W + \beta^M RV_t^M + \epsilon_{t+1} \tag{2.33}$$

where the realized volatilities over the various intervals are denoted RV_t^D (daily), RV_t^W (weekly) and RV_t^M (monthly).

Although far from an exhaustive review of the realized volatility literature, this section has highlighted the key developments of the field. In the following section, the theme of high frequency intraday data is continued, however the goal is modelling these intraday volatilities over the trading day. This is distinct from the concept of RV_t above, and is a topic highly relevant to the empirical work contained in this thesis.

2.4.5 Intraday Volatility

This section continues the theme of high frequency returns data, however here the objective is to effectively model the intraday volatility process over the trading day. These studies are often motivated by supposing a trading desk of a large institution requires up-to-date risk information (volatility forecasts) at small intervals throughout the trading day (see Engle and Sokalska, 2012). These forecasts are then used to set limit orders, for trade scheduling and risk management.

A well documented complication of modelling intraday volatilities is the diurnal or U-shaped pattern seen in volatility over the trading day, see Wood, McInish and Ord (1985) for perhaps the earliest discussion of this phenomena. Indeed, a successful univariate intraday volatility model needs to capture this diurnal pattern.

Andersen and Bollerslev (1997) presented a multiplicative component structure for use in this context,

$$r_{t,i} = \frac{h_t s_i \epsilon_{t,i}}{I^{1/2}} \qquad \epsilon_{t,i} \sim N(0,1) \tag{2.34}$$

where h_t is the daily volatility, s_i is the intraday diurnal pattern and $\epsilon_{t,i}$ is an error term. I denotes the number of intraday intervals over the trading day t. For s_i, Andersen and Bollerslev (1997) used a flexible Fourier functional form. They then filter the returns series for the estimated diurnal pattern, $\hat{s}_i$, using $\tilde{r}_{t,i} = r_{t,i}/\hat{s}_i$ and model the variance h_t

using a GARCH(1,1) specification like that in equation 2.20. The component structure of equation 2.34 is widely seen as the starting point for the intraday volatility literature.

Others have approached modelling intraday volatilities in a number of ways, often based on the Andersen and Bollerslev (1997) method. For example, Giot (2005a) modelled a deterministic diurnal pattern s_i, filtered the intraday returns and then compared various methods for the intraday variance component (including GARCH and RiskMetrics in equation 2.17). The aim of Giot (2005a) was to evaluate various intraday volatility frameworks in a market risk setting (using intraday Value at Risk), information useful to market participants such as traders and market makers.

The multiplicative component GARCH of Engle and Sokalska (2012) approached the issue from a similar angle, choosing to deal with each component of the univariate volatility process sequentially. In Engle and Sokalska (2012), the volatility is decomposed into daily, diurnal and intraday variances as

$$r_{t,i} = \sqrt{h_t s_i q_{t,i}}\, \epsilon_{t,i} \qquad\qquad \epsilon_{t,i} \sim N(0,1) \tag{2.35}$$

where h_t is the daily variance component, s_i the diurnal pattern over the trading day, $q_{t,i}$ the intraday variance, and, $\epsilon_{t,i}$ an error term. The estimation procedure involves modelling the daily variance, h_t, in the first instance, and then conditioning the intraday returns in order to estimate the diurnal pattern, s_i. The returns are then scaled by the diurnal component with a univariate GARCH capturing the remaining intraday persistence.

For the daily variance component, h_t, Engle and Sokalska (2012) used volatility forecasts, based on a multifactor risk model, that are commercially available for each company in their study. The intraday returns are scaled by the daily variances, allowing for the intraday diurnal pattern in the returns, s_i, to be modelled using

$$s_i = \frac{1}{T}\sum_{t=1}^{T} \frac{r_{t,i}^2}{h_t} \,. \tag{2.36}$$

The returns are then scaled by both the daily and diurnal variance components, denoted $z_{t,i}$,

$$z_{t,i} = \frac{r_{t,i}}{\sqrt{h_t s_i}} = \sqrt{q_{t,i}}\, \epsilon_{t,i} \ , \tag{2.37}$$

and the residual intraday variance modelled using a GARCH(1,1) specification

$$q_{t,i} = \omega + \alpha z_{t,i-1}^2 + \beta q_{t,i-1} \ , \tag{2.38}$$

where $\omega = (1 - \alpha - \beta)$. The usual constraints apply, that is $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta < 1$.

Engle and Sokalska (2012) addressed a shortcoming of the high frequency data, that is the case of illiquid stocks, in their dataset of 2721 U.S. equities. To overcome the difficulties illiquidity poses to parameter estimation, they discussed grouping equities and pooling the data (appending each returns series in the group to the end of the previous series). The intraday volatility model is then estimated for the pooled data. Results indicated that pooling leads to more stable estimates than individual modelling. Grouping companies by liquidity led to superior high frequency volatility forecasts for the illiquid stocks.

This section has overviewed the univariate modelling work underpinning the volatility forecasting literature. A natural extension of the univariate literature is to consider portfolios of two or more assets. Accordingly, the focus of this review turns to these multivariate models now.

2.5 Multivariate Time Series Forecasting

The previous section emphasised important developments in the univariate volatility modelling context. These models dealt with the estimation of the standard deviations contained on the diagonal of $\mathbf{D}_t$, in the popular decomposition of the conditional covariance matrix

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \tag{2.39}$$

where the covariance matrix, $\mathbf{H}_t$, is made up of the diagonal matrix of standard deviations, $\mathbf{D}_t$, and correlation matrix, $\mathbf{R}_t$. All are conditional on information up to time $t-1$. In this section, the concern is the conditional correlation matrix, or $\mathbf{R}_t$. The focus is how assets interact or comove with one another within a portfolio, with applications including portfolio allocation and risk management (see Section 2.2).

Unless stated otherwise, these models use so-called *volatility standardised* returns, $\hat{\epsilon}_{n,t}$, that is the returns series $r_{n,t}$ divided by the univariate standard deviation of asset n,

$\sqrt{h_{n,t}}$. This form of the volatility adjusted returns, $\hat{\epsilon}_{n,t} = r_{n,t}/\sqrt{h_{n,t}}$, is implied by the decomposition of the conditional covariance matrix $\mathbf{H}_t$ shown in equation 2.39. The form the conditional correlation matrix, $\mathbf{R}_t$, takes is restricted to the characteristics of a true correlation matrix. It must be symmetric with ones on the diagonal, positive definite and the elements of $\mathbf{R}_t$ are constrained such that $-1 < \rho < 1$. As will be discussed in this section, these requirements have proven to complicate matters for researchers.

A background of the developments in covariance modelling is now presented, covering the multivariate models relevant to the methodology presented in this document. See Andersen, Bollerslev, Christoffersen and Diebold (2006) for a comprehensive survey of these models, as well as material beyond the scope of this review.

2.5.1 Non-GARCH Methods

Simple to implement and practical, the methods outlined in this section require little to no optimisation and can be readily applied to a range of dimensions. The methods highlighted here are relevant to the empirical work in this thesis and in later chapters are classed as simple or *semi-parametric* models. This terminology refers to their comparative ease of implementation relative to the more complex multivariate GARCH specifications discussed in Section 2.5.3. Most are direct extensions of the univariate cases described in Section 2.4.1.

The most basic forecasting tool is a simple moving average (SMA). It is a popular tool of technical traders and investors in the multivariate setting due to its practical and computationally quick application. The SMA is shown as

$$\mathbf{H}_t = \frac{1}{K}\sum_{k=1}^{K} \hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k} \ , \tag{2.40}$$

where K is the moving average period (referred to as the *rolling window*), $\hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k}$ the kth lag of the outer product of volatility standardised returns, and $\mathbf{H}_t$ the forecast of the covariance matrix. Note here that the outer product of the standardised returns series, $\hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k}$, is used as the predictor of the covariance matrix. The resulting symmetric conditional covariance matrix should be positive definite as long as $N < K$ (Chiriac and Voev, 2011). A range of window lengths have been used in the literature, depending on the problem at hand. For example, the use of a full trading year (approximately 252 days) is consistent with Value at Risk (VaR) applications, in accordance with the Basel Committee on Banking Supervision (1996).

The exponentially weighted moving average (EWMA) of J.P. Morgan/Reuters (1996) RiskMetrics is a widely used extension of the SMA in equation 2.40. It places a higher weight on more recent observations. Fleming et al. (2001) extended the EWMA to a multivariate context, shown as

$$\mathbf{H}_t = \exp(-\alpha)\mathbf{H}_{t-1} + \alpha \exp(-\alpha)\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1} \; . \tag{2.41}$$

Here, $\exp(-\alpha)$ is the rate of decay, estimated using optimisation subject to the constraint $0 < \alpha < 1$ and $\hat{\boldsymbol{\epsilon}}_{t-1}$ is the volatility-adjusted return at time $t-1$. Fleming et al. (2001, 2003) examined the potential gain of volatility timing using the exponential weighting scheme in equation 2.41. Their reasoning was intuitive, that is if $\mathbf{H}_t$ is time varying, the covariance dynamics would be reflected in the path of the returns. Employing a method that requires the squares and outer products of the lagged returns was ideal for their purpose. Their choice of an exponential estimator was also well founded, as Foster and Nelson (1996) have shown the exponential weighting scheme will generally provide the smallest mean squared error (MSE). In addition, positive definiteness of the resulting $\mathbf{H}_t$ is assured. Fixing the parameter α avoids any optimisation to estimate the conditional covariances, however it can easily be obtained using standard estimation techniques. RiskMetrics provide a range of appropriate values of α for different data frequencies. For example, for equation 2.41 they suggest $\alpha = 0.06$ as the appropriate rate of decay for daily data.

The final non-GARCH style model examined here is the multivariate MIxed DAta Sampling (MIDAS) approach, an extension of Ghysels et al. (2006). Ghysels, Sinko and Valkanov (2007) alluded to extending the framework to the multivariate case as a natural path forward although they did not mention doing so for the purpose of covariance forecasting. León, Nave and Rubio (2007) proposed a bivariate MIDAS model to test of the empirical significance of the hedging component within a dynamic risk-return model. However, they did not extend the estimation of covariances beyond the two asset case. Broadly, this framework can be viewed as a different weighting scheme of past observations, comparative to the EWMA discussed previously. As discussed in the univariate context, the MIDAS approach has the flexibility of allowing the independent variables to be sampled at a higher (or lower) frequency than the dependent variable of interest. Many applications are possible, although discussion of this model in the multivariate context (and certainly in the large dimensional context) has been limited. In their review of the MIDAS framework and associated applications, Ghysels and Valkanov (2012) remarked that mixed data sampling in the multivariate context is a relatively new area of interest.

The MIDAS approach of Ghysels et al. (2006) in the multivariate case is shown as

$$\mathbf{H}_t = \bar{\mathbf{R}} + \sum_{k=0}^{K} \theta_k \hat{\boldsymbol{\epsilon}}_{t-k} \hat{\boldsymbol{\epsilon}}'_{t-k} \ . \tag{2.42}$$

$\mathbf{H}_t$ is the forecast of the conditional covariance matrix, $\bar{\mathbf{R}}$ the mean (unconditional sample) correlation, θ_k are the polynomial lag parameters, K is the maximum lag length and $\hat{\boldsymbol{\epsilon}}_{t-k} \hat{\boldsymbol{\epsilon}}'_{t-k}$ the forecasting variable. Similar to the univariate case, the weighting scheme used is often based on the Beta function although others have been used in the literature (see Engle et al., 2013, among others).

The methods outlined in this section are by no means an exhaustive list of the non-GARCH models used for forecasting the conditional covariance matrix, rather the necessary background for this research. The next section details the multivariate GARCH class of model, forming the basis of the research agenda presented in later chapters.

2.5.2 Multivariate Volatility Models

As in the univariate GARCH universe, the multivariate work is extensive and this section will touch the tip of a very large iceberg, aiming to provide a general background to this literature. The section highlights the development of this family of models, their limitations and provides an outline of the complexities of correlation forecasting (for recent surveys, see Bauwens, Laurent and Rombouts, 2006 and Silvennoinen and Teräsvirta, 2009).

Early multivariate models were direct extensions of the univariate GARCH family and are discussed in Section 2.4.2. The early multivariate GARCH (MGARCH) models enable the identification of common themes regarding the practical implementation of this class. Namely, the complications in developing models that meet the statistical requirement that the covariance matrix be positive definite, but also that the model be effectively parsimonious to avoid parameter proliferation when modelling the conditional covariance of multiple time series. This literature begins with the VECH specification of Bollerslev et al. (1988),

$$\mathrm{vech}(\mathbf{H}_t) = \mathbf{C} + \sum_{q=1}^{Q} \mathbf{A}_q \mathrm{vech}(\boldsymbol{r}_{t-q}\boldsymbol{r}'_{t-q}) + \sum_{p=1}^{P} \mathbf{B}_p \mathrm{vech}(\mathbf{H}_{t-p}) \ . \tag{2.43}$$

Here, vech($\cdot$) is an operator stacking the columns of the lower triangular part of the volatility matrix. $\mathbf{C}$ is a $N(N+1)/2 \times 1$ vector and $\mathbf{A}_q$ and $\mathbf{B}_p$ are parameter matrices, size $N(N+1)/2 \times N(N+1)/2$. The positive definiteness of the covariance matrix is not guaranteed by the VECH specification without further constraints, providing motivation for a restricted version that circumvents this issue.

The Baba Engle Kraft Kroner (BEKK) model of Engle and Kroner (1995) is

$$\mathbf{H}_t = \mathbf{C}\mathbf{C}' + \sum_{q=1}^{Q}\sum_{f=1}^{F} \mathbf{A}'_{f,q}(\boldsymbol{r}_{t-q}\boldsymbol{r}'_{t-q})\mathbf{A}_{f,q} + \sum_{p=1}^{P}\sum_{f=1}^{F} \mathbf{B}'_{f,p}\mathbf{H}_{t-p}\mathbf{B}_{f,p} \ . \tag{2.44}$$

The parameter matrices $\mathbf{A}_{f,q}$, $\mathbf{B}_{f,p}$ and $\mathbf{C}$ are $N \times N$. The matrix $\mathbf{C}$ is lower triangular. The summation to F governs the number of restrictions being imposed on the model (in comparison to the VECH in equation 2.43). The BEKK model achieves the first goal of positive definiteness of the covariance matrix. Subsequently, variations of the BEKK model have been used extensively in the literature. Despite this, the BEKK specification cannot be considered parsimonious, requiring N^2 parameters. The model quickly becomes very large for even a modest number of assets. As the modelling of large multivariate systems is an active pursuit for researchers, empirical work using these models has been limited in scope.

The Factor ARCH model of Engle, Ng and Rothschild (1990) was a notable advancement in the MGARCH literature. The dimensionality of the series is reduced by letting the dynamics of the N assets be determined by F common factors, explaining the covariation of the system. Based on the idea that a large portion of covariance in asset returns is driven by a set of common factors, the Factor ARCH model is a special case of the BEKK model above. Several extensions of this framework and similar have been suggested, see Bauwens, Laurent and Rombouts (2006) for a review.

2.5.3 Multivariate Correlation Models

The Constant Conditional Correlation (CCC) model of Bollerslev (1990) addressed the issue of parameter proliferation by decomposing the conditional matrix. Here, the conditional covariance matrix is

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R} \mathbf{D}_t \ . \tag{2.45}$$

The correlation matrix $\mathbf{R}$ is time invariant. The result is conditional covariances that are dynamic through time, however time dependent only on the variances (volatilities) of the individual assets. Despite the restriction of constant correlations, researchers including Laurent, Rombouts and Violante (2012) have found the CCC model to be useful in particular circumstances.

The Dynamic Conditional Correlation (DCC) model of Engle (2002),[12] is considered to be a parsimonious approach addressing both positive definiteness and parameter proliferation.[13] A generalisation of the Bollerslev (1990) CCC model, the DCC framework firstly estimates univariate GARCH models for each series to generate volatility standardised returns, $\hat{\epsilon}_{n,t} = r_{n,t}/\sqrt{h_{n,t}}$. Utilising the standard residuals obtained in the first instance, a so-called *pseudo* time varying correlation matrix $\mathbf{Q}_t$ is estimated. The pseudo-correlations are then scaled to form the conditional correlation matrix, $\mathbf{R}_t$. The specification used here is that of Aielli (2013), given by

$$\begin{aligned}
\mathbf{R}_t &= \text{diag}(\mathbf{Q}_\text{t})^{-1/2}\mathbf{Q}_\text{t}\text{diag}(\mathbf{Q}_\text{t})^{-1/2} \\
\mathbf{Q}_t &= \bar{\mathbf{Q}}(1-a-b) + a\ \text{diag}(\mathbf{Q}_{t-1})^{1/2}\hat{\boldsymbol{\epsilon}}_t\hat{\boldsymbol{\epsilon}}_t'\ \text{diag}(\mathbf{Q}_{t-1})^{1/2} + b\ \mathbf{Q}_{t-1}
\end{aligned} \tag{2.46}$$

where a and b are parameters subject to the constraints $a > 0$, $b > 0$ and $a + b < 1$, and $\hat{\epsilon}_{n,t} = r_{n,t}/\sqrt{h_{n,t}}$ the volatility standardised returns. The presence of the unconditional correlation matrix, $\bar{\mathbf{Q}}$, ensures the model is mean reverting (or correlation targeting) to the unconditional level. Persistence in volatility is demonstrated should the sum of the two parameters be close to unity, implying that the closer the sum is to one the more persistent

[12] See Engle and Sheppard (2001) for further discussion regarding the estimation of the DCC.

[13] Tse and Tsui (2002) have also proposed a similar framework, referred to as VC-GARCH. The DCC is described here in detail as it is the model used throughout the empirical work of this thesis.

the correlations. As the parameters here are scalar values, the correlation dynamics are equal for all assets. The literature points to the use of this model in place of the original and it is referred to as consistent DCC (cDCC).[14]

The coefficients of the cDCC model are estimated using a two stage quasi-maximum likelihood procedure. The log-likelihood is

$$\ln L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\mathbf{D}_t|) + \ln(|\mathbf{R}_t|) + \hat{\boldsymbol{\epsilon}}_t'\mathbf{R}_t^{-1}\hat{\boldsymbol{\epsilon}}_t\right) . \tag{2.47}$$

Recall the diagonal matrix of standard deviations is denoted $\mathbf{D}_t$, as in equation 2.39. The log-likelihood in equation 2.47 is included here to allude to the potential issue of inverting the potentially large dimensional correlation matrix $\mathbf{R}_t$ for this type of estimator. For standard maximum likelihood optimisation routines, this term will be computed for each t a number of times. For large N, inversion of this matrix becomes numerically intensive and will impact the practical implementation of any empirical application of this model. This point will be returned to and discussed at length later in this section.

Many applications of the cDCC exist in the literature. An interesting variation is the DCC-MIDAS of Colacito, Engle and Ghysels (2011), an extension of the GARCH-MIDAS touched on in the univariate discussion. This specification allows the daily, short-run, dynamics of the correlations to be governed by a DCC and coupled with a time varying long-run MIDAS regression.

The restriction placed on the correlation dynamics by two scalar parameters in equation 2.46 is perhaps unrealistic and relaxing this assumption has been considered. Engle (2002) generalises the above approach by altering $\mathbf{Q}_t$ in equation 2.46 to allow a and b to be $N \times N$ matrices denoted by $\mathbf{A}$ and $\mathbf{B}$ respectively,

$$\mathbf{Q}_t = \bar{\mathbf{Q}} \odot (\mathbf{1}_N - \mathbf{A} - \mathbf{B}) + \mathbf{A} \odot \hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}_{t-1}' + \mathbf{B} \odot \mathbf{Q}_{t-1} \tag{2.48}$$

where $\odot$ denotes element-by-element multiplication and $\mathbf{1}_N$ a $N \times N$ matrix of ones.

Similarly, Franses and Hafner (2009) suggest a Generalized DCC (GDCC) model in order to exploit the straightforward nature of the cDCC family, whilst allowing for differ-

[14] Aielli (2013) showed that the original DCC estimator is asymptotically biased, subsequently the specification originally put forward by Engle and Sheppard (2001) has been replaced in practice by equation 2.46.

ences in asset dynamics. The GDCC specification allows for differing dynamics between assets as in equation 2.48, that is

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - \bar{\alpha}^2 - \bar{\beta}^2) + \boldsymbol{\alpha}\boldsymbol{\alpha}' \odot \hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1} + \boldsymbol{\beta}\boldsymbol{\beta}' \odot \mathbf{Q}_{t-1} \ . \tag{2.49}$$

Here, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $N \times 1$ vectors of parameter values and $\bar{\alpha}$ and $\bar{\beta}$ are the average value of each vector respectively. Should $\alpha_1 = \cdots = \alpha_N$ and $\beta_1 = \cdots = \beta_N$, the resulting simplification is the cDCC model specified in equation 2.46. As $\mathbf{Q}_t$ is a weighted sum of positive semi-definite matrices (provided reasonable parameter values and $\bar{\mathbf{Q}}$ are used), the resulting $\mathbf{H}_t$ will be positive definite. Franses and Hafner (2009) made reference to the fact that the correlation targeting property of the cDCC model is lost in equation 2.49, as that would require use of $\bar{\mathbf{Q}} \odot (\mathbf{1}_N - \boldsymbol{\alpha}\boldsymbol{\alpha}' - \boldsymbol{\beta}\boldsymbol{\beta}')$, which is not positive definite. Estimation of the GDCC model is performed using the usual two step quasi-maximum likelihood process of the cDCC provided in Engle (2002).

The GDCC model of Franses and Hafner (2009) eases the strict restraints placed on the correlation dynamics of the assets within the portfolio, thus allowing better estimation of larger systems. However, dimensionality problems remain unsolved due to the computational burden of allowing for differing correlation dynamics among the assets. Intuitively, it is reasonable to consider a model where assets are grouped into blocks and the correlation dynamics between the blocks are allowed to differ. This way, the total number of assets in the system can be increased whilst somewhat retaining the effective parameterisation of the cDCC model. Billio, Caporin and Gobbo (2006) provided such a specification.

Consider the GDCC model in equation 2.49 and the $N \times N$ parameter matrices, $\boldsymbol{\alpha}\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}\boldsymbol{\beta}'$. By imposing a further restriction whereby the parameter matrices are block-diagonal with zeros in the off-diagonal blocks, the GDCC model becomes feasible for large systems. However, this Block Diagonal DCC does not allow for interaction between blocks, merely permitting the correlation dynamics of individual blocks to differ. Thus, the Block Diagonal DCC structure is simply a system of smaller block-sized DCC models. To seemingly address this issue Billio et al. (2006) presented an extension to the GDCC model and refer to this $\mathbf{Q}_t$ specification as Flexible-DCC. It is defined as

$$\mathbf{Q}_t = \mathbf{c}\mathbf{c}' \odot \bar{\mathbf{Q}} + \boldsymbol{\alpha}\boldsymbol{\alpha}' \odot \hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1} + \boldsymbol{\beta}\boldsymbol{\beta}' \odot \mathbf{Q}_{t-1} \ . \tag{2.50}$$

Here, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\mathbf{c}$ are partitioned vectors of B blocks similar to:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{B1} \\ \alpha_{B2} \\ \vdots \\ \alpha_{BN} \end{bmatrix} . \tag{2.51}$$

As described above, the correlation targeting property of the scalar-parameter cDCC is lost in equation 2.50. However, imposing the restriction $c_n c_m + \alpha_n \alpha_m + \beta_n \beta_m = 1$ for $n, m = 1, ..., N$, this property can be recovered as a special case.

Issues with blocking, including the optimal number of blocks and how best to allocate assets to blocks, have meant these models have not featured prominently in recent MGARCH literature. However, there appears to be two lines of argument for how to classify assets into blocks. The first is an economic view, that is assign blocks according to some economic criteria. A common example is industry classification, see Engle and Kelly (2012). This allows stocks within an industry to share a correlation whilst allowing for a different correlation between industries. The second point of view is data-driven and there is a growing body of work considering this, see Bonanno, Caldarelli, Lillo, Miccichè, Vandewalle and Mantegna (2004) and Matesanz and Ortega (2008) among others.

In addition to generalisations of the scalar dynamics of the cDCC to block-type structures, a range of extensions of the basic conditional correlation structure exists.[15] For example, the Regime Switching for Dynamic Correlations (RSDC) model of Pelletier (2006), as the name suggests, allows the conditional correlation matrix to switch between correlation regimes. Transition between the different states is governed by a Markov switching process, although the correlations stay constant when in a particular regime. Silvennoinen and Teräsvirta (2015) suggested a smooth transition between states, allowing the correla-

[15] Additionally, variations of the models presented in Section 2.5.2 and in this section have been suggested to account for asymmetry. See for example, Kroner and Ng (1998) and Cappiello, Engle and Sheppard (2006).

tions to vary smoothly between two extremes, driven by an observable transition variable. Regime switching in the univariate context was discussed in Section 2.4.3.

Alternative approaches have previously been suggested to cope with the problem of large dimensional systems, many based on factor-type methods similar to those discussed in Section 2.5.2. These models include the Principal Component Multivariate ARCH (see Ding, 1994 and Ding and Engle, 2001) and Orthogonal GARCH (see Alexander and Chibumba, 1997 and Alexander, 2002). The research shows that using the largest two principal components can account for at least 90% of the volatility of the entire system. Although empirically successful, the principal components methods have not garnered the popularity of the DCC-type MGARCH models discussed in this section.

The literature also discusses the drawbacks of the current conditional correlation models, namely cDCC. Pakel, Shephard, Sheppard and Engle (2014) suggested this family of models has dimensionality problems if N is large (that is, exceeds 50 or so assets). In contrast to imposing further structure on the system (for example, blocking), they introduced a *composite likelihood* approach to better estimate the conditional correlation matrix. This appears to lessen the difficulties imposed by this model.

The composite likelihood (CL) approach detailed here renders these methods plausible for large-scale applications, addressing the problem of bias afflicting the two step quasi-likelihood estimators. This approach has been used successfully in the mathematics literature for some time, see Lindsay (1988) and more recently, Varin and Vidoni (2005) for applications where standard likelihood methods are infeasible. The CL is constructed and maximised to provide the estimate of the covariance matrix. Pakel et al. (2014) provided evidence to suggest this method is attractive from both computational and statistical viewpoints. Further, they suggested the CL approach is appropriate in the case of more structured models such as multivariate factor models with time varying volatility.

The CL is the sum of quasi-likelihoods, obtained by breaking the portfolio of assets into subsets. Each subset yields a quasi-likelihood estimator, which can then be added to the others to produce the CL. The process avoids having to invert large covariance matrices, preventing burdensome computational issues[16] and also the bias introduced by an unknown incidental parameter.[17]

The CL procedure effectively transforms a vast dimensional system into a number of small ones. To do so, $\boldsymbol{r}_t$ is transferred into the data array $\mathbf{Y}_t = \{\mathbf{Y}_{1t}, ..., \mathbf{Y}_{Gt}\}$ where $\mathbf{Y}_{g,t}$ is a vector of small subsets of the data. This can be shown as $\mathbf{Y}_{g,t} = (r_{g_1,t}, r_{g_2,t})$, where $(g_1, g_2) \in \{1, \cdots, N\}^2$ and $g_1 \neq g_2$ for all $g = 1, \cdots, G$. Pakel et al. (2014) considered all unique pairs of data, therefore $G = N(N-1)/2$. Thus a valid quasi-likelihood for the gth subset is constructed to estimate the parameters. By averaging over a number of submodels and summing over the series a sample CL function is produced.

Evaluation of the CL costs $O(N^2)$ calculations, gaining an advantage over standard quasi-likelihood methods. In Monte Carlo simulations, the model performed comparatively better in terms of bias and RMSE than the standard quasi-likelihood estimator for both $N \to \infty$ and $T \to \infty$. The estimator can be $O(1)$ if necessary and remains unbiased even if the number of assets exceeds that of the observations.[18]

Engle and Kelly (2012) approached the issue of bias and computational burden differently. They investigated a time varying correlation model, Dynamic Equicorrelation (DECO), where all pairs of returns are restricted to have equal correlation on a given day. This equicorrelated matrix, $\mathbf{R}_t$ can be defined as

$$\mathbf{R}_t = (1 - \rho_t)\mathbf{I}_N + \rho_t \mathbf{1}_N \ . \tag{2.52}$$

Here, ρ_t is the equicorrelation, $\mathbf{I}_N$ the N-dimensional identity matrix, and $\mathbf{1}_N$ a $N \times N$ matrix of ones. The pairwise cDCC pseudo-correlations given in equation 2.46 are averaged

[16] A comparison of the computation (CPU) times for estimation of the original cDCC and cDCC-CL illustrates this point. For example, the time taken to estimate the second stage likelihood (that is, the correlation parameters a and b in equation 2.46) for the original cDCC is between 1.5 to 2 times longer than the cDCC using CL (all unique pairs). This is based on estimating the parameters once for portfolios of U.S. equities (daily returns data), with portfolio sizes of $N = 50$ to 100 assets and $T = 4200$.

[17] For further discussion on the topic of composite likelihood and the incidental parameter problem, see Neyman and Scott (1948), Lancaster (2000) and Pakel, Shephard and Sheppard (2009).

[18] The CL estimator will be consistent and asymptotically normal (proof provided by Pakel et al., 2014), as will the two stage estimation of Engle and Sheppard (2001). There will be some efficiency loss associated with use of the CL approach, however it is more robust to misspecification and bias than the original quasi-maximum likelihood approach (especially in high dimensions).

to form

$$\rho_t = \frac{1}{N(N-1)}\left(\mathbf{1}'_N \mathbf{R}_t^{cDCC}\mathbf{1}_N - N\right) = \frac{2}{N(N-1)}\sum_{n>m}\frac{q_{n,m,t}}{\sqrt{q_{n,n,t}q_{m,m,t}}} \tag{2.53}$$

where $q_{n,m,t}$ is the n, mth element of the pseudo-correlation matrix, $\mathbf{Q}_t$. Although a seemingly strong restriction on the dynamics of the correlation process, similar approaches have been applied throughout the financial literature. Additionally, the concept of equicorrelated matrices is not limited to financial applications. It has been applied in various fields due to its tractability for large dimensional problems, see for example Gill, Banneheka and Swartz (2005) and Leiva and Roy (2011). If required, the imposed structure of the equicorrelation matrix can be alleviated by blocking (discussed above). That is, the correlations between groups of assets are allowed to differ whilst having equal correlations between assets within a group, see Engle and Kelly (2012).

Equicorrelation circumvents the computational burden of the original cDCC model by simplifying the likelihood equation. To illustrate the difference between this specification and the cDCC estimator, consider the second step of the log-likelihood under DECO,

$$\begin{aligned} \ln L \quad = \quad & -\tfrac{1}{T}\textstyle\sum_{t=1}^{T}\Big[\ln\left([1-\rho_t]^{N-1}[1+(N-1)\rho_t]\right) \\ & +\tfrac{1}{1-\rho_t}\left(\textstyle\sum_{n=1}^{N}(\hat{\epsilon}_{n,t}^2) - \tfrac{\rho_t}{1+(N-1)\rho_t}\left(\textstyle\sum_{n=1}^{N}\hat{\epsilon}_{n,t}\right)^2\right)\Big] . \end{aligned} \tag{2.54}$$

Recall $\hat{\epsilon}_{n,t}$ are the returns adjusted for the first stage volatility estimates, $\hat{\epsilon}_{n,t} = r_{n,t}/\sqrt{h_{n,t}}$ and ρ_t given by equation 2.53.

The computation required under DECO is reduced to N-dimensional outer products with no matrix inversion or determinants as in cDCC. It is these T inversions and determinants that contribute to the cDCC framework being so burdensome it is impractical for vast systems. In contrast to the cDCC family, equicorrelated matrices have simple analytical determinants and inverses, ensuring optimisation and likelihood calculation are made feasible.[19]

[19] This discussion compares the original cDCC to the DECO model. Application of an estimation routine like composite likelihood addresses a large component of the computational differences between cDCC and DECO. There are however, efficiency costs associated with using a partial likelihood such as the composite likelihood scheme that the DECO does not suffer.

Another significant difference between the DECO and cDCC frameworks is the make-up of the correlation matrix, $\mathbf{R}_t$. In the cDCC model, an element of the correlation matrix, $\mathbf{R}_t^{cDCC}$, is the correlation of asset n and asset m at time t. It is dependent on the history of n and m. The same correlation for the DECO specification depends on the history of all pairs of assets in the system. The ability of DECO to pool information is conjectured to be the reason for the DECO model's forecasting superiority over the cDCC model, as reported in Engle and Kelly (2012). They offered results of Monte Carlo simulations where DECO outperformed composite likelihood cDCC in the equicorrelated case, although failed to do so for the non-equicorrelated process. However, the evidence presented suggests DECO performs better under misspecification than composite likelihood cDCC when comparing the two data generating processes.

This section has provided the highlights of an extensive literature surrounding the MGARCH class of covariance model. In the following section, the link between volatility and correlation is discussed.

2.5.4 Linking Volatility and Correlations

The previous sections have covered developments of the expansive GARCH literature relevant to this thesis, in both the univariate and multivariate contexts. Focus can now turn toward possible determinants of correlation, namely volatility. Empirically, the concept of a relationship between volatility and correlation is not new. Longin and Solnik (1995) modelled the conditional correlation between international markets. They found that international correlations were time varying, and rose during periods of high volatility. Similarly, Ramchand and Susmel (1998) found the correlations between the U.S. stock market and other world markets are on average 2 to 3.5 times higher when the U.S. market is in a high volatility state as opposed to a low volatility state. Bracker and Koch (1999) studied whether correlations between international equity markets are time varying as well as the economic reasoning behind it. They found volatility has a positive relationship with the magnitude of correlations. Erb, Harvey and Viskanta (1994) and Solnik, Boucrelle and le Fur (1996) argued world market volatility is a determinant of correlations across national markets, and Yang (2005) concluded correlations increase during periods of high market volatility.

The above studies have all considered the link between volatility and correlations in the context of international markets. An interesting example of the international setting and correlation is Europe. Several authors have studied the financial integration of members of the Eurozone (see Cappiello, Géhard, Kadareja and Manganelli (2006), Savva and Aslanidis (2012) and Taştan (2005) among others), and found increased integration over time. Integration is measured by increasing correlations between nations within the Eurozone. Work such as that of Christoffersen, Errunza, Jacobs and Jin (2014) has found evidence of an upward trend in global portfolio correlations over time. Although intuitive, these results may also be indicative of a global trend toward more integrated financial markets.

A natural extension of this empirical work is to develop models exploiting volatility as a determinant of correlations. Bauwens and Otranto (2013)[20] suggest several MGARCH-type models along these lines. They chose the cDCC model of Aielli (2013), Smooth Transition Conditional Correlation (STCC) of Silvennoinen and Teräsvirta (2015) and the Regime Switching for Dynamic Correlations (RSDC) model of Pelletier (2006) to extend in order to capture correlation dependence on market volatility. The VIX was used to proxy market volatility and a portfolio of U.S. equities is used to assess empirical performance of their Volatility Dependent class of model. Evidence of volatility as a determinant of correlations is found, specifically as a long-run effect. The Volatility Dynamic Conditional Correlation (VDCC) models of Bauwens and Otranto (2013) are highlighted here, as they are directly relevant to the empirical work in Chapter 4.

In the VDCC framework, the level of volatility can be included as an additive effect on the conditional pseudo-correlations by extending the specification of $\mathbf{Q}_t$ in equation 2.46 to be

$$\begin{aligned} \mathbf{Q}_t &= \bar{\mathbf{Q}}(1 - a - b - g\ \bar{v}_{t-1}) \\ &\quad + a\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t-1} \hat{\boldsymbol{\epsilon}}'_{t-1}\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} +\ b\ \mathbf{Q}_{t-1} + g\ v_{t-1}\ \mathbf{1}_N\ . \end{aligned} \tag{2.55}$$

Here, v_{t-1} is (VIX/100) at time $t-1$, $\bar{v}_{t-1}$ is the average of v up to $t-1$ and g a scaling parameter. This model is referred to as DCC-AVE. In the corresponding DCC-ARE model

[20] A paper of the same name has recently been published by Luc Bauwens and Edoardo Otranto. The published version appears in the *Journal of Business & Economic Statistics* (2016), Volume 34, Issue 2, pp. 254–268.

the regime of volatility is used, as in

$$\begin{aligned}\mathbf{Q}_t &= \bar{\mathbf{Q}}(1 - a - b - g\ \overline{\mathrm{E}_{t-1}(S_{t-1})}) \\ &\quad + a\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1}\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} +\ b\ \mathbf{Q}_{t-1} + g\ \mathrm{E}_{t-1}(S_{t-1})\ \mathbf{1}_N\ .\end{aligned} \tag{2.56}$$

Here, $\mathrm{E}_{t-1}(S_{t-1})$ is the probability of the high regime of the (VIX/100) at time $t-1$, $\overline{\mathrm{E}_{t-1}(S_{t-1})}$ is the average probability of the high regime from $t = 1$ to $t-1$ and g the relevant parameter.

To model the regime of the VIX, Bauwens and Otranto (2013) use a two state Markov switching autoregressive model, like that in equation 2.28. The log-likelihood function is assumed to be normal and the MS-AR(2) parameters are estimated using quasi-maximum likelihood estimation. Filtered one-step-ahead probabilities, updated at time t, are equal to the expected value of the regime, $\mathrm{E}_t(S_t) = \Pr(S_t = 1|\Psi_t)$, where Ψ_t denotes the information available. The expected value, $\mathrm{E}_{t-1}(S_{t-1})$, at time $t-1$ is used as the conditioning variable of the volatility regime in the relevant VDCC models.

Bauwens and Otranto (2013) also investigate volatility as having an indirect link with correlations through use of a non-linear effect. The correlation parameters a and b in equation 2.46 are allowed to be time varying and dependent on the level of the volatility. This model is referred to as the DCC-TVV model and can be specified

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - a_t - b_t) + a_t\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1}\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} + b_t\ \mathbf{Q}_{t-1} \tag{2.57}$$

where $a_t = a_0 + a_1 f_{a,t}$ and $b_t = b_0 + b_1 f_{b,t}$. For the function f a logistic specification is used

$$\begin{aligned} f_{a,t} &= 1/[1 + \exp^{-(\theta_{a,0}+\theta_{a,1}v_{t-1})}]\ , \\ f_{b,t} &= 1/[1 + \exp^{-(\theta_{b,0}+\theta_{b,1}v_{t-1})}]\ . \end{aligned} \tag{2.58}$$

The corresponding regime version of this model is DCC-TVR and can be expressed as

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - a_t - b_t) + a_t\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1}\ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} + b_t\ \mathbf{Q}_{t-1} \tag{2.59}$$

where $a_t = a_0 + a_1 f_{a,t}$, $b_t = b_0 + b_1 f_{b,t}$. Equation 2.58 is redefined as

$$\begin{aligned} f_{a,t} &= 1/[1 + \mathrm{e}^{-(\theta_{a,0}+\theta_{a,1}\mathrm{E}_{t-1}(S_{t-1}))}] \ , \\ f_{b,t} &= 1/[1 + \mathrm{e}^{-(\theta_{b,0}+\theta_{b,1}\mathrm{E}_{t-1}(S_{t-1}))}] \ . \end{aligned} \tag{2.60}$$

It should be noted that empirically Bauwens and Otranto (2013) found the parameter b to be constant, reducing the -TVV and -TVR models to only allow a_t to be dependent on volatility.

The VDCC framework detailed in equations 2.55 to 2.59 will result in a positive definite $\mathbf{Q}_t$ provided a reasonable $\bar{\mathbf{Q}}$ is chosen and the parameters are constrained appropriately (as in equation 2.46). For the case of DCC-AVE (equation 2.55) the constraints include $(a + b + g\bar{v}_{t-1} < 1)$. For DCC-ARE (equation 2.56) this becomes $(a + b + g\overline{\mathrm{E}_{t-1}(S_{t-1})} < 1)$. VDCC is easily applied to larger dimensions, however care needs to be taken to avoid computational issues during estimation (in the same way as the original cDCC model).

2.5.5 Realized Covariance and Intraday Covariance

The final concepts considered in this review are modelling realized covariances, discussed in Section 2.3.4, and the separate topic of intraday covariance. Recall the realized covariance,

$$RCOV_t = \widehat{\boldsymbol{\Sigma}}_t = \sum_{i=1}^{I} \boldsymbol{r}_{t,i}\boldsymbol{r}'_{t,i} \ . \tag{2.61}$$

In the multivariate setting, a number of papers seek to model correlations at a given frequency, often daily, by using higher frequency intraday returns. Termed realized covariance ($RCOV$), the use of intraday data to generate daily correlation matrices has gained popularity in the literature. Distinct to this is the study of the high frequency covariances and correlations over the trading day, referred to in this thesis as *intraday covariance* or *intraday correlation*. This section will discuss $RCOV$ before attention turns to intraday covariance, the subject of direct relevance to this thesis.

Numerous techniques have been suggested to model realized covariances. Chiriac and Voev (2011) employed Cholesky factorization to build the realized covariance matrix. Others apply the Wishart distribution (see Gourieroux, Jasiak and Sufana, 2009 for an introduction to the Wishart distribution), for example Golosnoy, Gribisch and Liesenfeld

(2012) proposed the Conditional Autoregressive Wishart (CAW) model for the analysis of realized covariance matrices of asset returns. Their model can be estimated by maximum likelihood and satisfies the requirement of positive definiteness. Jin and Maheu (2013) suggested point forecasts can be improved by using a joint component model of returns and $RCOV$ based on Wishart distributions. Further, Bonato, Caporin and Ranaldo (2012) considered a restricted specification, with a view toward lessening problems with dimensionality, that performs favourably compared to the full model.

The large dimensional context is investigated by Hautsch, Kyj and Malec (2015), based on earlier work of Hautsch, Kyj and Oomen (2012). Their method of regularisation and blocking constructs a covariance matrix from a set of smaller matrices. They grouped assets trading at similar frequencies and scaled the resulting covariance matrix so that it is positive definite and well-conditioned. This technique dealt with the complications that arise from market microstructure effects such as bid-ask spread and allowed for data to be sampled more frequently than methods that employ synchorized intervals.

The DECO framework of Section 2.5.3 (see equation 2.53) has been extended to exploit the use of high frequency data to form equicorrelation forecasts, see Clements, Coleman-Fenn and Smith (2011), Bauwens et al. (2014), and Aboura and Chevallier (2015). Clements et al. (2011) suggested a realized equicorrelation measure (to forecast daily equicorrelation)

$$REC_t = \frac{2}{N(N-1)} \sum_{n>m} \frac{RCOV_{n,m,t}}{\sqrt{RCOV_{n,n,t} RCOV_{m,m,t}}} . \tag{2.62}$$

Here, $RCOV_{n,m,t}$ is the n, mth element of the realized covariance matrix in equation 2.61 for a given day t. They found that use of realized equicorrelation leads to superior portfolio outcomes over equivalent models using daily returns only.

Intraday covariance is distinct to the techniques above and is yet to see an explosion in terms of research volume. It concerns the use of high frequency data collected at small intervals throughout the trading day to model pairwise intraday covariance dynamics. The near-continuous flow of price and trade data presents researchers with opportunities, as well as unique challenges, to capture the dynamics of multivariate systems. Motivations for understanding these processes as they evolve throughout the trading day are varied, with applications such as hedging (see Frey, 2000), temporal trading strategies and the impact of news arrival (see Goodhart and O'Hara, 1997), among numerous others. Modelling intraday volatilities of individual assets has proven popular in recent work and an overview of this literature is contained in Section 2.4.5 of Chapter 2. Engle and Sokalska (2012) offer a univariate model and note that using their component model technique is optimal when pooling the returns data of a number of stocks. However, despite developments in the modelling of intraday volatility processes in the univariate case, few papers have studied modelling multivariate dynamics at high, intraday, frequencies.

Some authors have noted a pattern in the correlations evident over the trading day, for example Allez and Bouchaud (2011) and Tilak, Széll, Chicheportiche and Chakraborti (2013). These papers used eigenvector decompositions of the correlation matrix to study the dynamics of correlations over the trading day for U.S. equities. Allez and Bouchaud (2011) documented the average correlation increased over the trading day, however they did not model these effects. Existence of patterns in the intraday correlations leads to questions about how to model and subsequently forecast these dynamics. This topic is of particular interest for the research presented in Chapter 5.

2.6 Evaluating Covariance Forecasts

Given the sheer number of possibilities for estimating and forecasting the covariance matrix, model selection and ranking is an important consideration for anyone undertaking a portfolio allocation exercise. Indeed, the accuracy and efficacy of the forecast is of economic importance. Many of the models described here adequately capture the characteristics of volatility and covariance discussed earlier and provide reasonable forecasts in the large dimensional setting. How to choose between them for the purposes of portfolio allocation is an important issue and has been the subject of discussion in the literature. The unobservable nature of volatility, discussed in Section 2.3.2, requires that a suitable proxy be selected for comparison to the resulting forecast. This section highlights this issue by outlining popular techniques for covariance forecast selection, focusing on methods used in later chapters, and directly relates it to the multivariate and portfolio allocation context.

2.6.1 Loss Functions

Any evaluation of volatility and covariance forecasts, like those generated from the models outlined in the previous sections, require specification of some loss function. In response to the importance of assessing the accuracy of volatility forecasts extensive surveys regarding loss functions have been undertaken, see Hansen and Lunde (2006), Patton (2011) and Doolan (2011). This review does not seek to provide an exhaustive summary of such functions, rather to highlight the key points from this literature.

A loss function, either statistical in nature such as the mean squared error (MSE), mean absolute error (MAE) and quasi-likelihood function (QLK), or an economic measure such as the global minimum variance (GMV) portfolio, needs to fulfil two basic criteria. Firstly, it should reach a minimum when the forecast equals the actual volatility and secondly, apply an increasing penalty as the forecast error increases. If these two conditions are violated, the resulting rank may not be consistent with the true ranking order of the volatility forecasts.

In their most basic univariate specifications, the loss functions above can be shown as

$$\text{MSE} : \mathcal{L}(h_t, \hat{\sigma}_t^2) = (h_t - \hat{\sigma}_t^2)^2 \tag{2.63}$$

$$\text{MAE} : \mathcal{L}(h_t, \hat{\sigma}_t^2) = |h_t - \hat{\sigma}_t^2| \tag{2.64}$$

$$\text{QLK} : \mathcal{L}(h_t, \hat{\sigma}_t^2) = \ln \hat{\sigma}_t^2 + \frac{h_t}{\hat{\sigma}_t^2} \ , \tag{2.65}$$

where $\hat{\sigma}_t^2$ is the volatility proxy and h_t the forecast. Each appear to satisfy both of the conditions outlined, however recent work by Patton (2011) found that the presence of noise in the volatility proxy means only MSE and QLK provide consistent ranking of competing forecasts. Hansen and Lunde (2006) discussed sufficient conditions under which a loss function is consistent, that is unaffected by noise in the volatility proxy, and referred to inconsistency as objective bias. They found objective bias declines as noise does, rather than declining as T increases.

Multivariate extensions of the statistical loss functions, where $\widehat{\boldsymbol{\Sigma}}_t$ is the covariance proxy and $\mathbf{H}_t$ the forecast, can be expressed as

$$\text{MSE} : \mathcal{L}(\mathbf{H}_t, \widehat{\boldsymbol{\Sigma}}_t) = \frac{1}{N^2}\text{vec}(\mathbf{H}_t - \widehat{\boldsymbol{\Sigma}}_t)'\text{vec}(\mathbf{H}_t - \widehat{\boldsymbol{\Sigma}}_t) \tag{2.66}$$

$$\text{MAE} : \mathcal{L}(\mathbf{H}_t, \widehat{\boldsymbol{\Sigma}}_t) = \frac{1}{N^2}\mathbf{1}'|\mathbf{H}_t - \widehat{\boldsymbol{\Sigma}}_t|\mathbf{1} \tag{2.67}$$

$$\text{QLK} : \mathcal{L}(\mathbf{H}_t, \widehat{\boldsymbol{\Sigma}}_t) = \ln|\mathbf{H}_t| + \text{vec}(\mathbf{H}_t^{-1} \odot \widehat{\boldsymbol{\Sigma}}_t)'\mathbf{1}\ . \tag{2.68}$$

Here, vec$(\cdot)$ represents the column stacking operator, $|\cdot|$ represents the absolute value operator and $\mathbf{1}$ a vector of ones. Despite there only being $N(N+1)/2$ unique elements in the covariance matrix, all N^2 elements are compared in the MSE and MAE.

A different approach is that of the GMV portfolio, an economic loss function. The GMV portfolio (risky assets only) with weights $\boldsymbol{w}_t$ is solved with the solution

$$\boldsymbol{w}_t = \frac{\mathbf{H}_t^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{H}_t^{-1}\mathbf{1}}\ . \tag{2.69}$$

The loss function is then

$$\mathcal{L}(\boldsymbol{w}_t, \widehat{\boldsymbol{\Sigma}}_t) = \boldsymbol{w}_t'\ \widehat{\boldsymbol{\Sigma}}_t\ \boldsymbol{w}_t\ . \tag{2.70}$$

Becker et al. (2015) show the GMV portfolio variance is consistent,

$$\frac{\partial \boldsymbol{w}_t'\widehat{\boldsymbol{\Sigma}}_t\boldsymbol{w}_t}{\partial\hat{\sigma}_t} = \text{vech}(\boldsymbol{w}_t\boldsymbol{w}_t') \tag{2.71}$$

and

$$\frac{\partial^2 \boldsymbol{w}_t'\widehat{\boldsymbol{\Sigma}}_t\boldsymbol{w}_t}{\partial\hat{\sigma}_t\partial\hat{\sigma}_t'} = \mathbf{0} \longrightarrow \frac{\partial^3 \boldsymbol{w}_t'\widehat{\boldsymbol{\Sigma}}_t\boldsymbol{w}_t}{\partial\hat{\sigma}_t\partial\hat{\sigma}_t'\partial h_{k,t}} = \mathbf{0} \quad \forall\, k\ . \tag{2.72}$$

Here, vech$(\cdot)$ is the lower triangle column stacking operator, $h_{k,t}$ is the kth element of vech$(\mathbf{H}_t)$ and $\mathbf{0}$ is a zero matrix.

There are a variety of loss functions available for assessing the accuracy of volatility forecasts. The work of Hansen and Lunde (2006), Patton (2011) and Becker et al. (2015), among others, highlights the care that needs to be taken in selection of ranking tools. The next section discusses the Model Confidence Set, a test of predictive ability designed to assess the significance of differences between competing forecasts.

2.6.2 The Model Confidence Set

The Model Confidence Set (MCS) proposed by Hansen, Lunde and Nason (2011), is used to evaluate the significance of any differences in performance between models. Unlike other tests of predictive ability, such as the superior predictive ability (SPA), reality check (RC) and equal predictive accuracy (EPA) test, the MCS does not require setting a benchmark model to which other specifications are compared.

The MCS begins with a full set of candidate models $\mathcal{M}_0 = 1, ..., m_0$ and sequentially discards elements of $\mathcal{M}_0$ to achieve a smaller set of models. This Model Confidence Set will contain the best model with a given level of confidence $(1-\alpha)$. A loss function is denoted $\mathcal{L}(\mathbf{H}_t, \widehat{\mathbf{\Sigma}}_t)$ and the resulting loss differential between models i and j at time t is

$$d_{i,j,t} = \mathcal{L}(\mathbf{H}_t^i, \widehat{\mathbf{\Sigma}}_t) - \mathcal{L}(\mathbf{H}_t^j, \widehat{\mathbf{\Sigma}}_t) \ , \qquad i, j = 1, ..., m_0 \ . \tag{2.73}$$

The procedure involves testing the following

$$H_0 \ : \ \mathbb{E}(d_{i,j,t}) = 0 \ , \qquad \forall \ i > j \in \mathcal{M} \tag{2.74}$$

for a set of models $\mathcal{M} \subset \mathcal{M}_0$. The initial step sets $\mathcal{M} = \mathcal{M}_0$. The t-statistic, $t_{i,j}$,

$$t_{i,j} = \frac{\bar{d}_{i,j}}{\sqrt{\widehat{\text{var}}(\bar{d}_{i,j})}} \ , \qquad \bar{d}_{i,j} = \frac{1}{T}\sum_{t=1}^{T} d_{i,j,t} \tag{2.75}$$

scales the average loss differential of models i and j by $\widehat{\text{var}}(\bar{d}_{i,j})$. The estimate of the variance of average loss differential can be obtained using the bootstrap procedure in Hansen et al. (2011).

These $(m_0 - 1)m_0/2$ t-statistics are converted into one test statistic using

$$T_R = \max_{i,j \in \mathcal{M}} \frac{|\bar{d}_{i,j}|}{\sqrt{\widehat{\text{var}}(\bar{d}_{i,j})}} \ . \tag{2.76}$$

It is referred to as the range statistic, with rejection of the null hypothesis occurring for large values of the statistic. The worst performing model, determined by

$$i = \arg\max_{i \in \mathcal{M}} \frac{\bar{d}_{i,j}}{\sqrt{\widehat{\text{var}}(\bar{d}_{i,j})}} \,, \qquad \bar{d}_{i,j} = \frac{1}{m-1} \sum_{j \in \mathcal{M}} \bar{d}_{i,j,t} \tag{2.77}$$

is removed from $\mathcal{M}$ and the entire procedure repeated on the new, smaller set of models. Iterations continue until the null hypothesis is not rejected, and the resulting set of models is the MCS, denoted $\hat{\mathcal{M}}_\alpha$.

Use of the MCS is becoming more common in empirical applications. Examples in multivariate settings similar to those contained in later chapters include Laurent et al. (2012) and Becker et al. (2015), among others.

2.6.3 Economic Value

In the context of portfolio allocation, practical considerations of forecasting and economic value further the discussion of evaluating competing covariance forecasts. The portfolio theory discussed in Section 2.2.1 allows computation of the vector of optimal portfolio weights, assuming a target portfolio return of μ_0,

$$\hat{\boldsymbol{w}}_t = \frac{\mathbf{H}_t^{-1}\mu}{\mu' \mathbf{H}_t^{-1}\mu} \, \mu_0 \,. \tag{2.78}$$

Here, μ is the vector of expected returns.

The methodology of Fleming, Kirby and Ostdiek (2001, 2003) formed a relative measure of economic value that has become popular in the empirical portfolio allocation literature. The Fleming Kirby Ostdiek (FKO) method involves computing the relative economic benefit of each of the forecasts of the covariances by forming optimal portfolios (using the weights in equation 2.78) and finding the constant δ that solves

$$\sum_{t=T_0}^{T_1} U(r_{p,t}^1) = \sum_{t=T_0}^{T_1} U(r_{p,t}^2 - \delta) \,. \tag{2.79}$$

Here $r_{p,t}^1$ and $r_{p,t}^2$ are the portfolio return series of two competing methods of forecasting and T_0 and T_1, respectively mark the beginning and the end of the forecast period. The constant δ is the incremental value of using the second method instead of the first and measures the maximum average daily return an investor would forgo to switch to the second forecasting method. The investor's utility function is assumed to be

$$U(r_{p,t}) = -\exp(-\lambda\, r_{p,t}) \tag{2.80}$$

that is negative exponential utility (Skouras, 2007), where λ is the investor's risk aversion coefficient and $r_{p,t}$ is their return during the period to time t.

Following the method of Fleming, Kirby and Ostdiek (2001, 2003) block bootstrapping can be used to generate artificial samples of returns to minimise the uncertainty around the expected returns required for the formation of the optimal portfolios. This requires that samples of observations are generated, using randomly selected blocks of random length (with replacement) from an empirical dataset of asset returns. In practice, a range of bootstrap lengths, target portfolio returns and risk aversion coefficients are used to support the robustness of findings.

Fleming, Kirby and Ostdiek (2003) also touch on extending the study of the value of volatility timing to longer forecast horizons. They consider the gains of volatility timing over horizons of one week up to one year. Given that changes occur for both the static and volatility-timing portfolios, the relative portfolio performance does not change. Furthering the discussion of longer forecast horizons, the suitability of the MIDAS regression framework (discussed in Sections 2.4.1 and 2.5.1) should be emphasised. Insofar as portfolio rebalancing in practice may take place at longer horizons than daily, the mixed frequency approach of methods such as MIDAS can be readily applied.

The stability of the portfolios can be considered a useful proxy for any economic value differences between the competing methods, without the need to make any assumptions regarding transaction costs. Clements and Silvennoinen (2013) removed the consideration of transaction costs from their analysis and compared absolute change in FKO optimal portfolio weights, linking competing forecasts and portfolio stability. Their measures included the mean absolute change in portfolio weight over the X bootstraps,

$$|\Delta_w| = \frac{1}{X} \sum_{x=1}^{X} \frac{1}{T_1 - T_0 - 1} \sum_{t=T_0+1}^{T_1} |w_t - w_{t-1}| \tag{2.81}$$

and the standard deviation of weight changes

$$\sigma_{\Delta_w} = \frac{1}{X}\sum_{x=1}^{X}\sigma_{w_t - w_{t-1},x} \ . \tag{2.82}$$

A similar technique to that described in equation 2.81 is used in Chapter 3 to evaluate competing forecasts.

This section has emphasised important developments in the evaluation of volatility and covariance forecasts, the final piece of any review concerned with forecasting volatility and correlations. The following section concludes this chapter and briefly previews the empirical work contained in the remainder of the thesis.

2.7 Conclusion

This chapter highlighted the findings and work of others in the area of volatility and correlation timing, portfolio allocation and forecasting. The importance of modelling the common characteristics of volatility processes has been emphasised, setting the backdrop for empirical applications undertaken in the following chapters. The univariate and multivariate modelling techniques relevant to this dissertation have been summarised and the methods used to evaluate forecasts in previous applications set out. Clearly, the ideal way of dealing with large correlation matrices remains the subject of ongoing interest for researchers and is of practical importance for finance practitioners. Additionally, how to effectively model high frequency intraday correlation dynamics is also an open, and important, question. Research into generating intraday correlations (and covariances) is not nearly as extensive as in the case of univariate intraday volatility.

Chapter 3 is the first of three empirical chapters contained in this thesis. It provides the setting for the practical nature of this research agenda, emphasising the usefulness of an assumption of equicorrelation in the management of a portfolio of equities. Chapter 3 also provides the framework for how this research plans to address the question of modelling large correlation matrices in Chapters 4. The final empirical application addresses the idea of intraday correlation dynamics from an MGARCH perspective. The comprehensive look at the MGARCH framework provided in this thesis seeks to build on the work outlined in this review. Further, the thesis contributes to the existing correlation modelling literature, concerned specifically with the modelling correlations of a portfolio of financial assets.

Chapter 3

On the Benefits of Equicorrelation for Portfolio Allocation

3.1 Introduction and Motivation

This chapter considers the performance of several correlation forecasting models, all appropriate for use in large dimensions. The aim is to assess whether relatively complex models such as the multivariate GARCH (MGARCH) framework lead to superior portfolio outcomes compared to simpler, moving average based methods. The models are evaluated across a range of portfolio sizes to provide insights into the value of the correlation forecasts in the large portfolio allocation context. The MGARCH methods used to generate forecasts of the correlation matrix include the Dynamic Equicorrelation (DECO) model of Engle and Kelly (2012), the consistent Dynamic Conditional Correlation (cDCC) model of Aielli (2013) and Constant Conditional Correlation (CCC) model of Bollerslev (1990). The moving average based models include both simple and exponentially weighted moving averages and the MIxed DAta Sampling (MIDAS) of Ghysels et al. (2006). These simpler methods are classed as *semi-parametric* as no correlation parameters are estimated. Each has been discussed in Chapter 2. A small simulation study is presented to assess the

A paper of the same name has been published from the research contained in this chapter, co-authored with Adam Clements and Annastiina Silvennoinen. The published version appears in the *Journal of Forecasting* (2015), Volume 34, Issue 6, pp. 507–522.

behaviour of the cDCC and DECO methods under known data generating processes. Empirically, the global minimum variance (GMV) portfolio and Model Confidence Set (MCS) are used to compare all methods. Portfolio weight stability and relative economic value are also considered. Out-of-sample periods are divided into subsamples based on the relative level of volatility, and performance again compared as before. Given the market turbulence of recent years, it is interesting to investigate these differing volatility conditions and any potential impact on the forecasting performance of models.

Others have focused on the evaluation of multivariate GARCH-type models, see Laurent et al. (2012) and Caporin and McAleer (2012). However, this chapter differs from these works in two important ways. First, the sole use of daily data as opposed to intraday allows scope for larger dimensional portfolios (the largest number of assets here is 100). Daily data allows a number of issues posed by the use of high frequency data for large dimensional problems, such as stock liquidity problems, to be circumvented. Any problems with the positive definitiveness of the covariance matrix are also easily avoided. Secondly, a wider range of non-MGARCH-based methods are considered here, shifting the focus to a more practical, less GARCH-orientated study.

Evidence is found in favour of assuming equicorrelation across various portfolio sizes, particularly during times of crisis. During periods of market calm the suitability of the constant conditional correlation model cannot be discounted, especially for large portfolios. The results indicate that the assumption of equicorrelation offers stability (both from a portfolio exposure perspective and in terms of minimising portfolio variance). It is conjectured that the reduced estimation error of the DECO methodology provides superior forecasts. On balance, DECO outperforms cDCC for periods of both market tranquillity and turbulence in the context of minimising portfolio variance, especially in higher dimensions. However, the key difference between the two models is stability of portfolio weights. The equicorrelated model produces forecasts that lead to comparatively stable portfolio exposures. It is relatively immune to the increase in change in weights seen for all other models over the forecast period as portfolio size increases. In terms of the incremental gain of switching from one particular model to another, DECO dominates the other models across the various portfolio sizes. These findings further strengthen the economic value argument in favour of equicorrelation.

3.2 Methodology

3.2.1 Generating Forecasts of the Correlation Matrix

This section outlines the models used to forecast the conditional correlation matrix. Recall from Chapter 2 the decomposition of the covariance matrix, popularised by Engle (2002),

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \ . \tag{3.1}$$

The conditional covariance matrix is denoted $\mathbf{H}_t$, $\mathbf{R}_t$ is the conditional correlation matrix and $\mathbf{D}_t$ is the diagonal matrix of conditional standard deviations of the returns at time t. This decomposition leads naturally to an estimation procedure that can be split into two stages. A univariate volatility model is used to form conditional standard deviations. The conditional correlations are then estimated (or generated without estimation of parameters in the case of the non-MGARCH methods) using asset returns standardised by volatility estimates in $\mathbf{D}_t$. Consistent with the MGARCH literature, correlation forecasts are generated for each model outlined here using this two stage procedure.

The standard deviations of the individual assets in $\mathbf{D}_t$ are estimated using the univariate GARCH model of Bollerslev (1986) in equation 2.20 and GJR-GARCH model of Glosten et al. (1993) in equation 2.22. The variance forecasts, $h_{n,t}$, are used to adjust the returns series, $r_{n,t}$, to form the volatility standardised returns, $\hat{\epsilon}_{n,t} = r_{n,t}/\sqrt{h_{n,t}}$.

Attention now turns to estimating the conditional correlations. Firstly, the Constant Conditional Correlation (CCC) model of Bollerslev (1990) assumes that the conditional correlations are constant over time, that is $\mathbf{R}_t = \mathbf{R}$ in equation 3.1. As described in Section 2.5.3 of Chapter 2, any variation in $\mathbf{H}_t$ is a result of variation in $\mathbf{D}_t$. The correlation matrix, $\mathbf{R}$, is formed by calculating the sample correlations of the volatility standardised returns, $\hat{\epsilon}_{n,t}$, generated by estimating univariate GARCH models for each series.

An extension of CCC is the consistent Dynamic Conditional Correlation (cDCC) model of Aielli (2013). A so-called *pseudo* time varying correlation matrix $\mathbf{Q}_t$ is estimated using the $T \times N$ matrix $\hat{\boldsymbol{\epsilon}}$, as

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1-a-b) + a \ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t-1} \hat{\boldsymbol{\epsilon}}'_{t-1} \ \mathrm{diag}(\mathbf{Q}_{t-1})^{1/2} + b \ \mathbf{Q}_{t-1} \ . \tag{3.2}$$

Recall from Chapter 2 that $\bar{\mathbf{Q}}$ is the unconditional sample correlation of volatility standardised returns. The parameters a and b are subject to the positivity constraints $a > 0$, $b > 0$ and $a + b < 1$, and $\hat{\boldsymbol{\epsilon}}_{t-1}$ the vector of volatility standardised returns. As the parameters here are scalar values, the correlation dynamics are the same for all assets. The pseudo-correlation matrix $\mathbf{Q}_t$ in equation 3.2 is transformed into the conditional correlation matrix, $\mathbf{R}_t$, using equation 2.46.

The cDCC is considered to be a parsimonious MGARCH model, requiring estimation of two parameters to form the correlation matrix for the entire portfolio of assets. However, as discussed in Section 2.5.3, the form of the likelihood (see equation 2.47) presents the computationally burdensome issue of inverting a potentially large correlation matrix. To enable the cDCC framework to be tractable for the high dimensions used in this chapter, estimation of the parameters, a and b, uses the composite likelihood (CL) approach of Pakel et al. (2014) (see Section 2.5.3). This renders the cDCC plausible for large-scale applications by effectively dividing the large problems into a number of subsets, in this chapter all unique pairs of data are used. A quasi-likelihood is estimated for each of these pairs and then added to the others to form the CL. By avoiding the inversion of large correlation matrices, the CL approach allows cDCC to be compared to the alternative correlation forecasting models which are more tractable in large dimensions.

The third MGARCH approach of forecasting the correlation matrix is the DECO or equicorrelation approach of Engle and Kelly (2012), discussed at length in Section 2.5.3. The equicorrelated model defines the conditional correlation matrix $\mathbf{R}_t$ as containing ones on the diagonal and the equicorrelation term ρ_t as the off-diagonal elements. Recall from the previous chapter that all pairs of returns are restricted to have this equal correlation, the ρ_t, on a given day t. To calculate the equicorrelation, DECO averages the pairwise cDCC pseudo-correlations (the elements of $\mathbf{Q}_t$ given in equation 3.2). DECO is computationally quicker to estimate than the cDCC framework as assuming equicorrelation simplifies the likelihood equation (see equation 2.54), circumventing the inversion of $\mathbf{R}_t$.[2]

In addition to using the MGARCH methods outlined, a number of simple methods are used to capture correlations for multivariate systems, not assuming equicorrelation. These

[2]In terms of computation time, estimating the cDCC using the CL approach allows it to be comparable to estimating a DECO. However, DECO avoids any potential costs incurred by the cDCC-CL's use of a partial likelihood and the inversion of $\mathbf{R}_t$ altogether. It is therefore reasonable to expect some benefit beyond simple time savings can be contributed to the DECO model.

models are of practical interest due to the ease with which they can be implemented. In the forms presented here, they require no second stage estimation of parameters to form the conditional correlation matrix. Details regarding these moving average based methods have been provided in Section 2.5.1 of Chapter 2. Each model is used to generate a pseudo-correlation matrix, $\mathbf{Q}_t$, and this is rescaled to give the correlation matrix, $\mathbf{R}_t$, using equation 2.46.

The most basic forecasting tool is a simple moving average (SMA),

$$\mathbf{Q}_t^{SMA} = \frac{1}{K}\sum_{k=1}^{K} \hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k} \tag{3.3}$$

where K is the moving average period, and $\hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k}$ the kth lag of the outer product of the volatility standardised returns. The requirement for positive definiteness is $K > N$. A 252-day rolling window is used (this corresponds to a trading year) to ensure the covariance matrix is positive definite. The use of a full trading year is also consistent with value-at-risk (VaR) applications, in accordance with the Basel Committee on Banking Supervision (1996).

The exponentially weighted moving average (EWMA) of Fleming et al. (2001) places a higher emphasis on more recent observations than the SMA. It can be shown as

$$\mathbf{Q}_t^{EWMA} = \exp(-\gamma)\mathbf{Q}_{t-1}^{EWMA} + \gamma \exp(-\gamma)\hat{\boldsymbol{\epsilon}}_{t-1}\hat{\boldsymbol{\epsilon}}'_{t-1} \ . \tag{3.4}$$

The rate of decay, $\exp(-\gamma)$, is set using $\gamma = 2/(K+1)$, where $K = 252$, following the window length of the SMA described above.

The third method used is the MIxed DAta Sampling (MIDAS) model of Ghysels et al. (2006),

$$\mathbf{Q}_t^{MIDAS} = \bar{\mathbf{Q}} + \sum_{k=0}^{K} b(k, \boldsymbol{\theta})\hat{\boldsymbol{\epsilon}}_{t-k}\hat{\boldsymbol{\epsilon}}'_{t-k} \ , \tag{3.5}$$

where parameters $\theta = [\theta_1, \theta_2]'$ govern the beta density weighting scheme $b(k, \boldsymbol{\theta})$. The maximum lag length K is again set to 252 days. The parameter θ_1 is restricted to equal 1 and $\theta_2 = 0.98$, implying a slow decay.[3] The MIDAS framework is popular for a range

[3]See the RiskMetrics Technical Document (1996) for further discussion regarding the length of the window and a comparison of optimal decay rates.

of applications although most focus on univariate implementation of the model. Thus the use of the MIDAS in this context remains a largely open area of research.

Both moving average techniques and the MIDAS approach outlined above are simplistic in nature and are readily applied to a range of dimensions. In the forms described above they require no optimisation at all. Together with the first step of conditional volatility estimation, each model can be thought of as a semi-parametric approach.

3.2.2 Evaluating Forecasts

The previous section detailed the models used to forecast the covariance matrix, $\mathbf{H}_t$. This section will discuss the methods of evaluating such forecasts. In the first instance, the volatility of global minimum variance (GMV) portfolios is compared in order to evaluate forecasting performance of the competing models. Statistical significance of any differences in the GMV portfolio volatilities is examined using the MCS of Hansen et al. (2011). A measure of portfolio stability is generated by considering the average absolute change in portfolio weights. This study of the economic value of the correlation forecasting methods is furthered by quantifying and comparing the incremental gain of switching from a particular model to another. For completeness, an equally weighted portfolio (EQ-W) is also generated as an example of a portfolio where no forecasting or volatility timing is used.

The benefits of utilising the GMV portfolio as the loss function for this problem center on not needing to specify or make assumptions regarding the expected return of the portfolio. Both Caporin and McAleer (2012) and Becker et al. (2015) employ the GMV portfolio as a useful tool to compare correlation forecasts. The GMV portfolio made up of risky assets only satisfies

$$\min_{\boldsymbol{w}_t} \boldsymbol{w}_t' \, \mathbf{H}_t \, \boldsymbol{w}_t \, , \tag{3.6}$$

subject to $\mathbf{1}'\boldsymbol{w}_t = 1$, with weights

$$\boldsymbol{w}_t = \frac{\mathbf{H}_t^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{H}_t^{-1}\mathbf{1}} \, . \tag{3.7}$$

Once the GMV portfolios are formed given the forecasts from each of the models, an annualised percentage volatility for each is calculated and compared.

The MCS is used to evaluate the significance of any differences in performance between models. The MCS is discussed in Section 2.6.2, so only a brief description is provided here. Use of the MCS in similar multivariate settings includes Becker et al. (2015) and Laurent et al. (2012), among others. The premise of the MCS procedure is to avoid specifying a benchmark model. It instead begins with a full set of candidate models and sequentially discards members, leaving a set of models exhibiting equal performance. This MCS will contain the best model with a given level of confidence $(1 - \alpha)$. Here, the GMV portfolio-based loss function is defined as

$$\mathcal{L}(\mathbf{H}_t) = \boldsymbol{w}_t' \mathbf{r}_t \mathbf{r}_t' \boldsymbol{w}_t \; . \tag{3.8}$$

The loss differential between two competing models over the time series $t = 1, \; ..., \; T$ is calculated and the null hypothesis in equation 2.74 is tested for the set of models. The t-statistic in equation 2.75 scales the average loss differential of the two models by the variance of average loss differential, $\widehat{\text{var}}(\bar{d}_{i,j})$. The estimate of $\widehat{\text{var}}(\bar{d}_{i,j})$ is obtained using the bootstrap procedure in Hansen et al. (2011). These t-statistics are converted into one test statistic, the range statistic, defined in equation 2.76. Rejection of the null hypothesis occurs for large values of the statistic. The worst performing model is removed from the set and the entire procedure repeated on the new, smaller set of models. Iterations continue until the null hypothesis is not rejected, the resulting set of models is the MCS.[4]

In terms of analysing forecasts, transactions costs are an important practical issue. Without assuming a specific form for the transactions costs, forecasts can be ranked in terms of the stability of the portfolio weights they generate. This is measured by the absolute weight changes for each portfolio. The equally weighted portfolio is not included in this analysis. The absolute percentage weight change at time t for a given asset n is calculated as $|\Delta w_{n,t}| = |(w_{n,t} - w_{n,t-1})/w_{n,t-1}|$. Stability is measured by calculating the median absolute weight change for each asset in a portfolio, n, and taking the average across the N assets as

$$\mu_{MED} = \sum_{n=1}^{N} \text{median}(|\Delta w_{n,t}|)/N \; . \tag{3.9}$$

[4]An alternative MCS statistic known as the semi-quadratic (SQ) measure was also calculated, however is not reported. In general, the range statistic contained here and those reported in later chapters provided a more conservative MCS (that is, a larger set). For this reason the range statistic was chosen for the empirical analysis. The SQ results are omitted for brevity and are available on request.

In addition to consideration of portfolio stability, the economic value of the correlation forecasting methods is also compared using the methodology of Fleming et al. (2003). That is, the relative economic benefit of each of the forecasts of the correlations is measured by forming optimal portfolios and finding the constant δ that solves equation 2.79 in Chapter 2. The constant δ is the incremental value of using the second method instead of the first. It measures the maximum average daily return an investor would forgo to switch to the second forecasting method. The investor's utility function is assumed to be negative exponential utility (see equation 2.80). Following the method of Fleming et al. (2003), block bootstrapping is used to generate artificial samples of returns to take into account the uncertainty surrounding the expected returns required for the formation of the optimal portfolios. These samples are 5000 observations in length (samples of 10000 were also used; however, this did not lead to substantive differences in results) and are generated by randomly selecting blocks of random length, with replacement, from the original sample. A bootstrap is considered acceptable if the expected return is positive.[5] The bootstrapping procedure is repeated 500 times with δ calculated for each replication.

3.3 Costs and Benefits of DECO

A simulation study is carried out to assess the behaviour of the forecasting methods when the data generating process (DGP) is known to be either equicorrelated (DECO) or non-equicorrelated (cDCC). Of particular interest here is the question of whether there is a cost of incorrectly assuming equicorrelation for the purposes of portfolio allocation. Portfolios of $N = 5, 10, 25, 50, 100$ are generated with $T = 2500$ and 1000 simulations are carried out in each case.[6]

The DGP return vector is

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \tag{3.10}$$

where $\boldsymbol{\epsilon}_t$ are innovations that follow a multivariate normal distribution, $\Phi(0, \mathbf{H}_t)$, and $\boldsymbol{\mu}$ is assumed to be 0. The conditional covariance matrix can be decomposed as $\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t$ as in equation 2.39. This decomposition leads to the returns in equation 3.10 being gen-

[5]The positivity requirement is necessary as the portfolios are made up of risky assets, therefore the expected return of each portfolio generated is assumed to be positive.

[6]For each replication, samples of 4500 are generated and the first 2000 observations discarded as the lead-in period.

erated in two steps. First, the vector of randomly generated innovations are correlated using either a cDCC or DECO process, see Chapter 2 and in particular equations 2.46 and 2.52. For the first time step, $t = 1$, the conditional correlation matrix is set to be the unconditional correlation, $\bar{\mathbf{Q}}$. This matrix is estimated from an empirical dataset of U.S. equities, see Section 3.4 of this chapter for details. The second step of the DGP produces conditional correlated heteroscedastic returns using a GJR-GARCH(1,ϕ,1) model, see equation 2.22. The parameters of the GJR-GARCH model are set to be empirically reasonable values and different for each of the simulated returns series. Note also that the simulated data is generated using the original cDCC model (in the case of the DGP being the cDCC), whereas composite likelihood (CL) is used for estimation (hence cDCC-CL).

The initial in-sample period is 2000 observations, giving 500 one-step-ahead forecasts. The correlation model parameters are re-estimated every 20 observations, approximating a trading month[7]. Results presented here have been averaged over the 1000 simulations. Table 3.1 contains the mean and standard deviations, averaged across the simulations, of the correlation parameters in equation 3.2 for each estimation method. In general, there appears to be a large cost associated with using cDCC-CL to estimate an equicorrelated process. However, the use of DECO in the case of a non-equicorrelated process bears little, if any, cost in estimation accuracy. It in fact provides more accurate mean estimates for large values of N. This supports the use of DECO regardless of whether the underlying correlation process is equicorrelated or a standard cDCC, especially in large multivariate systems. In the equicorrelated case, cDCC-CL provides poor estimates of the parameter a and appears to be approaching zero as portfolio size increases. Unsurprisingly, overall DECO provides the better parameter estimates under the assumption of an equicorrelated process. The DECO parameter estimates display a higher standard deviation than those of the cDCC-CL, with the exception of the b parameter in the equicorrelated process. The results of this study support the similar findings of Engle and Kelly (2012) and further highlight the usefulness of equicorrelation.

[7]The re-estimation period is shorter in the empirical example than the 20 observations used for simulations, see Section 3.5. The longer window in simulations is simply to reduce computation time and does not impact the results.

		$a = 0.05$			
DGP	N	cDCC-CL	DECO	cDCC-CL	DECO
cDCC		$\bar{x}$		s.d.	
	5	0.0504	0.0510	0.0008	0.0020
	10	0.0504	0.0512	0.0005	0.0019
	25	0.0503	0.0507	0.0003	0.0016
	50	0.0502	0.0503	0.0003	0.0015
	100	0.0502	0.0502	0.0003	0.0014
DECO					
	5	0.0167	0.0503	0.0008	0.0020
	10	0.0108	0.0504	0.0005	0.0017
	25	0.0081	0.0499	0.0004	0.0012
	50	0.0079	0.0497	0.0003	0.0009
	100	0.0074	0.0497	0.0003	0.0007

		$b = 0.9$			
DGP	N	cDCC-CL	DECO	cDCC-CL	DECO
cDCC		$\bar{x}$		s.d.	
	5	0.8961	0.8932	0.0020	0.0054
	10	0.8967	0.8940	0.0012	0.0051
	25	0.8967	0.8960	0.0008	0.0040
	50	0.8968	0.8970	0.0007	0.0036
	100	0.8970	0.8976	0.0006	0.0036
DECO					
	5	0.9205	0.8939	0.0058	0.0051
	10	0.9236	0.8955	0.0058	0.0045
	25	0.9255	0.8980	0.0052	0.0031
	50	0.9243	0.8987	0.0045	0.0022
	100	0.9262	0.8993	0.0042	0.0017

Table 3.1: Mean ($\bar{x}$) and standard deviation (s.d.) of correlation parameter values for each DGP and estimation method, averaged across the 1000 simulations.

3.4 Data

The portfolios used contain a selection of S&P 500 stocks that continuously traded over the period 3 January 1996 to 31 December 2012. The full dataset contains 100 stocks and 4271 observations. All GICS sectors are represented across the dataset and the full list of stocks, including their ticker code, company name and sector, is provided in Appendix A. Over 60% of the assets contained in the dataset represent the Industrials, Consumer Staples, Consumer Discretionary and Health Care sectors. Log returns are calculated using $r_{n,t} = \log p_{n,t} - \log p_{n,t-1}$, where $p_{n,t}$ denotes the daily closing price of asset n at time t, adjusted for stock splits and dividends.

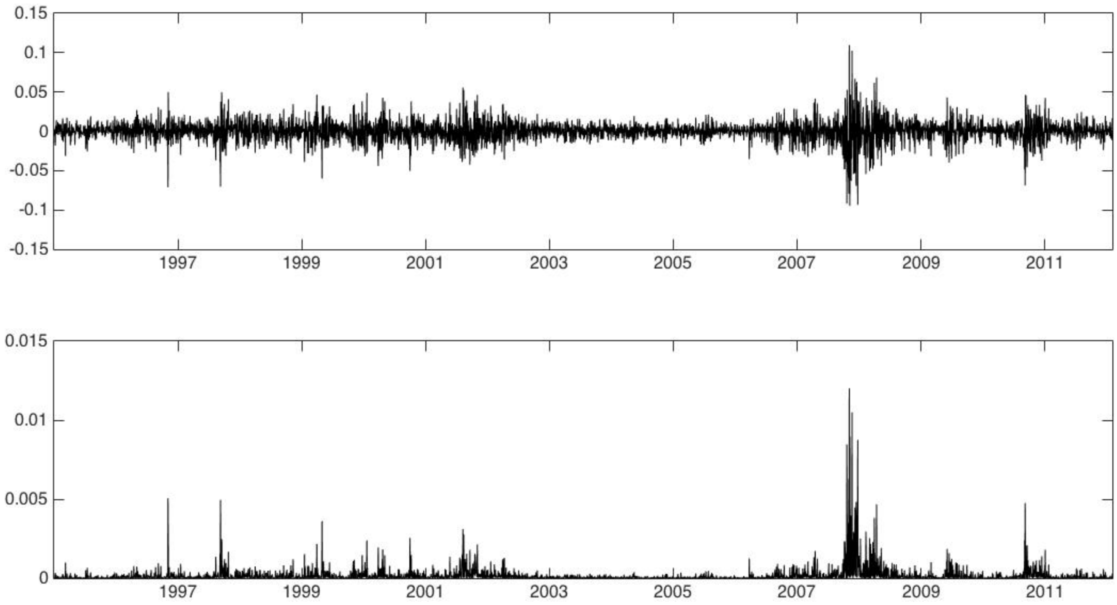

Figure 3.1: Daily returns, $\boldsymbol{r}_t$, of the S&P500 index (top) and squared daily returns of the index (bottom). Period spans 3 January 1996 to 31 December 2012.

Descriptive statistics for each stock are provided in Appendix A. The upper panel of Figure 3.1 shows the S&P 500 returns series and the lower panel the squared index returns series. Of note are the periods of relative high and low volatility over the sample. It is becoming increasingly common for researchers to evaluate forecasting methods for sub-periods of differing levels of volatility, see Luciani and Veredas (2015) for a recent example. The beginning of the sample is characterised by relatively low volatility, followed by a higher overall level of volatility that continues until the end of the in-sample period of 2000 observations. The high volatility spans a period from around mid 1997 until late 2003. This period includes events such as the dot-com bubble and September 11. The following three or so years are again a time of lower market volatility. From around March 2007 is a period of higher overall volatility corresponding to the onset of the global financial crisis (GFC). Finally, the last portion of the sample is a period of lower relative volatility. These changes are of interest as this chapter considers possible effects the overall level of volatility might have on the relative performance of the forecasting methods.

3.5 Empirical Results

This section contains the results of the empirical study, outlining the evaluation of the correlation forecasts described earlier. Portfolios used here contain $N = 5, 10, 25, 50, 100$ assets, randomly chosen from the list of 100 stocks of the S&P 500 (available in Appendix A). The forecasting horizon is one day. The in-sample period is 2000 observations, allowing for 2271 one-step-ahead forecasts. An expanding window is used for estimation and correlation parameters are re-estimated every 5 observations (approximating a trading week).[8]

The significance of the GJR–GARCH asymmetry parameter ϕ is tested to avoid any potential cost associated with unnecessary estimation. Nine stocks are found to have insignificant (at the 5% level) asymmetry coefficients, ϕ, subsequently their volatility processes are estimated using GARCH, as suggested by Thorp and Milunovich (2007). The remaining 91 stocks' univariate volatility processes are estimated using GJR–GARCH. One-step-ahead forecasts of the correlation matrix, $\mathbf{R}_{t+1}$, are generated using the MGARCH and semi-parametric approaches discussed in Section 3.2.1.

N	EQ-W	MIDAS	SMA	EWMA	CCC	cDCC	DECO
5	23.6612	17.7930	18.3648	17.5828	18.1969	17.3645	17.0460
10	28.6109	17.5249	18.1462	17.3154	17.8250	16.9898	16.9813
25	21.7895	13.8291	14.1532	13.7366	13.5186	13.2511	12.9283
50	20.6940	13.7027	13.6887	13.5402	12.8916	12.9794	12.7979
100	21.1951	13.9067	14.0765	13.4768	12.2585	12.7043	12.5088

Table 3.2: Annualised percentage volatility of out-of-sample minimum variance portfolio returns for each volatility timing strategy. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

Results presented in Table 3.2 are the out-of-sample standard deviations of the GMV portfolio returns described above, across the various portfolio sizes and models. As expected, the equally weighted portfolio results in a higher standard deviation across all portfolio sizes. The DECO method provides the lowest measure of volatility across each of the portfolios with the exception of the largest ($N = 100$), where CCC provides the lowest standard deviation. For the largest portfolio DECO delivers the second lowest standard

[8] The re-estimation period is shorter in the empirical example than the 20 observations used for simulations, see Section 3.3. The longer window in simulations is simply to reduce computation time and does not impact the results.

deviation after CCC, with cDCC producing the third lowest. The cDCC method follows DECO for the small and moderate portfolio sizes, providing the second lowest standard deviations for $N = 5$, 10 and 25. Of the semi-parametric methods, the EWMA method results in comparatively lower measures of volatility for all portfolio sizes and is followed by MIDAS in each case, with the exception of $N = 50$, where SMA follows EWMA. Although inferior to the MGARCH models, the semi-parametric methods perform relatively well for the small portfolios. However, the gap in performance widens as portfolio size increases.

N	EQ-W	MIDAS	SMA	EWMA	CCC	cDCC	DECO
5	0.0220	0.0880*	0.0880*	0.1540*	0.0880*	0.1540*	1.0000*
10	0.0070	0.0070	0.0070	0.0120	0.0120	0.9740*	1.0000*
25	0.0001	0.0001	0.0001	0.0001	0.1200*	0.3510*	1.0000*
50	0.0000	0.0000	0.0000	0.0000	0.8600*	0.7470*	1.0000*
100	0.0000	0.0000	0.0130	0.0000	1.0000*	0.0380	0.5570*

*Table 3.3: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.*

While the results in Table 3.2 provide simple rankings, the MCS is used to statistically distinguish between the performance of the models and these results are presented in Table 3.3. The MCS contains the best model(s), with a level of confidence of 95%. Unsurprisingly, the equally weighted portfolio is excluded from the MCS for all N. In the case of $N = 5$, all other models are included in the MCS. For the moderate portfolios the cDCC is also contained in the MCS along with DECO. DECO is the only method included in the MCS across all portfolio sizes and CCC is the only other model included for the largest portfolio. This is similar to the results of Laurent et al. (2012), although this study takes the analysis further in considering various portfolio sizes and indeed larger N. DECO is thought to exhibit less estimation error relative to the cDCC model as N increases and this may account for its performance in this setting. In terms of the cDCC method, as portfolio size increases the estimation error dominates due to the necessary estimation of the unconditional correlation matrix; see Ledoit and Wolf (2004) for discussion of estimation error and the sample covariance matrix. Equicorrelation has previously been found to be useful as a shrinkage target by Ledoit and Wolf (2004) and its usefulness is also apparent here in the portfolio allocation context.

Period	N	EQ-W	MIDAS	SMA	EWMA	CCC	cDCC	DECO
Dec 2003 to Feb 2007 (Low 1)								
2001:2806	5	15.0031	13.0022	13.0952	12.9613	12.8362	12.8828	12.8636
	10	15.9680	11.2053	11.8570	11.1706	11.3528	11.1181	11.2110
	25	11.7705	9.7725	10.3012	9.5927	9.3855	9.2998	9.3500
	50	10.8722	9.3769	9.7177	9.2217	8.8776	8.8158	9.0778
	100	10.5383	9.6070	9.3852	9.0828	8.2113	8.3618	9.0089
Mar 2007 to Dec 2011 (High)								
2807:4019	5	29.3304	21.3296	22.1953	21.0212	22.0561	20.7357	20.2280
	10	35.9006	21.6395	22.3349	21.3349	22.0780	20.8946	20.8495
	25	27.5080	16.6972	16.9906	16.6209	16.4495	16.0252	15.5058
	50	26.2766	16.5644	16.5242	16.4153	15.6727	15.7803	15.2207
	100	27.0432	16.6911	17.2704	16.3221	14.9781	15.5267	14.8674
Dec 2011 to Dec 2012 (Low 2)								
4020:4271	5	13.5321	10.9012	10.8591	10.9010	10.6220	10.7072	10.8686
	10	18.8057	10.6127	10.8008	10.6055	10.2632	10.2466	10.2399
	25	13.8606	8.7718	8.7683	8.7888	7.9360	8.3913	8.3728
	50	12.5644	9.5454	8.5559	9.1692	7.9942	8.4713	9.9033
	100	13.1667	10.3757	8.3107	9.5653	7.6932	8.4336	9.3437

Table 3.4: Annualised percentage volatility of out-of-sample minimum variance portfolio returns for each volatility timing strategy, split into high and low volatility. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

To gain a deeper understanding of forecast performance, the out-of-sample period has been split into periods of relatively high and low volatility. In this dataset, lower relative volatility is seen during the beginning and end of the out-of-sample period. The period of higher volatility corresponds approximately to the GFC of 2007/2008, beginning February 2007 with a higher overall level of volatility through to the end of 2011.[9] The annualised percentage volatilities of GMV portfolio returns in Table 3.4 show overall patterns similar to the full sample results in Table 3.2. The equally weighted portfolio is inferior in all cases. The MGARCH methods dominate the less complex models for all portfolios across each of the subsamples of high and low volatility. It is the CCC forecasts that provide a lower standard deviation for all portfolios, except that of 10 assets, during the second period of low volatility.

During the first period of low volatility, the MGARCH forecasts provide the smallest standard deviations, with cDCC and DECO outperforming the simpler methods across

[9]These subsamples are unequal lengths to provide an accurate representation of the general level market volatility in the out-of-sample period, in particular the post-GFC period of low volatility.

all portfolios, with the exception of $N = 10$. For this portfolio, DECO performs poorly in comparison to the other models; however, this is an isolated case. For CCC, the results are mixed across the various portfolios. Of the semi-parametric models, EWMA provides the least volatility, followed by MIDAS, for all portfolio sizes except the largest. For the second low volatility period, representing the post-GFC period, the results are mixed across the various portfolio sizes. For the small portfolio, $N = 5$, CCC provides the smallest standard deviation, followed by cDCC and SMA. EWMA and MIDAS are equivalent in terms of volatility for $N = 5$. For the moderate portfolio sizes, the MGARCH methods dominate the less complex models. Most notable for this post-GFC period of lower relative volatility is the performance of CCC for the large portfolios. The CCC model appears to provide an adequate forecast of correlation for the larger portfolios as it provides the lowest standard deviation across all methods. This is in contrast to the sometimes poorer performance of this method for the pre-GFC period of lower volatility. Overall, CCC performs well during periods of market tranquility across various portfolio sizes, as does cDCC.

During the period of high volatility DECO provides the lowest annualised percentage volatility for all N, suggesting the assumption of equicorrelation may be of benefit during times of crisis. As is the case for the total sample period (Table 3.2), the dominance of DECO appears to increase with N. The CCC method performs comparatively badly to the other methods for the small portfolio sizes; however, the reverse is true for the large portfolios. In these cases, the EWMA dominates SMA and MIDAS and the CCC model is superior to cDCC. Across the small and moderate portfolios cDCC follows DECO.

Table 3.5 contains the MCS results for the high and low volatility subsamples and they are broadly consistent with the full sample results in Table 3.3. The size of the MCS differs between that of the entire out-of-sample period and each subsample. For the smallest portfolio, $N = 5$, all models with the exception of the equally weighted portfolio are included in the MCS across all sub-periods. The cDCC is included in the MCS along with CCC for the largest portfolio in the first low volatility period; however, it is excluded from the MCS for $N = 100$ during the second low volatility subsample. During this period the CCC model dominates for $N = 50$ and 100, and is included in the MCS for all portfolios. In line with previous discussion, DECO is included with a p-value of 1 during the high volatility period across all N. This means it is the least worst method, that is

Period	N	EQ-W	MIDAS	SMA	EWMA	CCC	cDCC	DECO
Dec 2003 to Feb 2007 (Low 1)								
2001:2806	5	0.0230	0.0820*	0.0820*	0.1220*	1.0000*	0.8710*	0.9060*
	10	0.0000	0.3080*	0.0000	0.4350*	0.0540*	1.0000*	0.4350*
	25	0.0000	0.0001	0.0000	0.0020	0.4880*	1.0000*	0.7350*
	50	0.0000	0.0000	0.0000	0.0000	0.4800*	1.0000*	0.3910*
	100	0.0000	0.0000	0.0000	0.0000	1.0000*	0.1520*	0.0000
Mar 2007 to Dec 2011 (High)								
2807:4019	5	0.0240	0.0740*	0.0740*	0.1760*	0.0740*	0.1760*	1.0000*
	10	0.0040	0.0040	0.0040	0.0130	0.0130	0.8530*	1.0000*
	25	0.0000	0.0000	0.0000	0.0000	0.0700*	0.3000*	1.0000*
	50	0.0040	0.0040	0.0040	0.0040	0.6980*	0.6980*	1.0000*
	100	0.0000	0.0000	0.0190	0.0000	0.8700*	0.0880*	1.0000*
Dec 2011 to Dec 2012 (Low 2)								
4020:4271	5	0.0000	0.0710*	0.0710*	0.0710*	1.0000*	0.3760*	0.0710*
	10	0.0000	0.2070*	0.0170	0.2380*	0.9970*	1.0000*	0.9970*
	25	0.0000	0.0001	0.0001	0.0001	1.0000*	0.0400	0.0580*
	50	0.0000	0.0000	0.0190	0.0000	1.0000*	0.0230	0.0000
	100	0.0000	0.0000	0.0040	0.0000	1.0000*	0.0000	0.0000

*Table 3.5: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.*

the method that would be excluded last from the MCS. These results broadly support those of Laurent et al. (2012), although here it is found that during periods of relative market tranquility the performance of cDCC is sample specific, especially in the case of the largest portfolios. Indeed, during these periods the CCC outperforms the other MGARCH specifications and this seems to confirm their findings. Evidence is also found supporting the assumption of equicorrelation during periods of crisis, a method unexplored by Laurent et al. (2012).

N	MIDAS	SMA	EWMA	CCC	cDCC	DECO
5	0.1007	0.0777	0.0990	0.0814	0.0934	0.0953
10	0.1398	0.1223	0.1429	0.1185	0.1420	0.1365
25	0.1984	0.1589	0.1938	0.1736	0.1981	0.1457
50	0.2427	0.1912	0.2404	0.2157	0.2379	0.1510
100	0.2521	0.2143	0.2442	0.2120	0.2356	0.1453

Table 3.6: Average, μ_{MED}, of the median absolute change in portfolio weights across each model for the out-of-sample period. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

Table 3.6 contains the median absolute change in portfolio weights, μ_{MED} in equation 3.9, for each model across the entire out-of-sample period. It is used here to measure the stability of the global minimum variance portfolios formed using each of the various methods of correlation forecasts. The overall trend is increasing instability of portfolio weights as N increases, although μ_{MED} drops slightly across all methods as the portfolio size increases from $N = 50$ to $N = 100$. As N increases, DECO performs comparatively better than all other models in terms of this measure of stability. All other methods, including CCC and cDCC, are comparatively much more volatile in terms of portfolio weights over the forecast period as N increases. For the small portfolios of $N = 5$ and 10 the SMA and CCC methods provide the smallest values of μ_{MED} respectively. DECO provides a more stable portfolio in terms of asset weights for the moderate and large portfolios, with cDCC providing relative instability. From an economic point of view, the relative instability of the CCC and cDCC forecasts provide further evidence in favour of equicorrelation. Christoffersen et al. (2014) mention that the dominance of DECO can be attributed to the somewhat 'noisy' estimates of the pairwise correlations provided by cDCC and this is confirmed here in terms of portfolio allocation.

Similar results are obtained when taking into account periods of relatively high and low volatility (Table 3.7). The advantage of assuming equicorrelation is evident as DECO provides the most stable weights across the various portfolios, regardless of the subsample. The CCC method provides mixed results, although it provides stability during periods of market calm for large portfolio sizes. The cDCC method is broadly much more volatile than DECO in terms of portfolio weights regardless of the sub-period. Of the semi-parametric methods, the results are mixed, although SMA appears more stable in terms of weights as N increases, and this is the case regardless of subsample. As N increases, DECO again appears more stable in comparison to all other approaches. This is perhaps indicative of it containing less estimation error in the forecasts of the correlation matrix.

Tables 3.8 to 3.12 report the average value of the constant δ in equation 2.79, a measure of the relative economic value of choosing a particular correlation forecasting method over another, for each of the various portfolio sizes. Optimal portfolios (risky assets only) are formed using block bootstrapping to minimise the uncertainty around expected returns, by taking artificial samples of random length from the original dataset (with replacement).

Period	N	MIDAS	SMA	EWMA	CCC	cDCC	DECO
Dec 2003 to Feb 2007 (Low 1)							
2001:2806	5	0.0797	0.0769	0.0800	0.0691	0.0728	0.0723
	10	0.0959	0.1007	0.0997	0.0918	0.1055	0.0944
	25	0.1638	0.1553	0.1567	0.1627	0.1834	0.1416
	50	0.2120	0.1989	0.2133	0.2167	0.2229	0.1502
	100	0.2144	0.2156	0.2160	0.2090	0.2191	0.1542
Mar 2007 to Dec 2011 (High)							
2807:4019	5	0.1167	0.0802	0.1176	0.0963	0.1100	0.1163
	10	0.1864	0.1446	0.1897	0.1426	0.1767	0.1736
	25	0.2327	0.1687	0.2338	0.1954	0.2165	0.1718
	50	0.2753	0.1990	0.2718	0.2270	0.2610	0.1654
	100	0.2872	0.2235	0.2714	0.2263	0.2581	0.1564
Dec 2011 to Dec 2012 (Low 2)							
4020:4271	5	0.1232	0.0937	0.1302	0.0887	0.1334	0.1319
	10	0.1581	0.1454	0.1693	0.1529	0.1756	0.1711
	25	0.2068	0.1877	0.2077	0.1652	0.1948	0.1303
	50	0.2429	0.1960	0.2361	0.2066	0.2295	0.1423
	100	0.2696	0.2151	0.2545	0.2067	0.2243	0.1321

Table 3.7: Average, μ_{MED}, of the median absolute change in portfolio weights across each model, split into periods of high and low volatility. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

Here a positive value represents the economic gain of choosing the method in each column over that in each row, with the proportion of bootstraps where δ is positive reported in small text underneath. Results reported assume an expected return of 6% and a risk aversion coefficient of $\lambda = 2$.[10] Expected returns of 8% and 10%, as well as a risk aversion coefficient of $\lambda = 5$, were also used; however, this did not lead to any qualitative difference in the results.

As expected, the equally weighted portfolio is inferior to all methods for all portfolio sizes. Broadly in line with the evaluation presented previously, DECO dominates the other forecasts in all cases. Overall, differences between models become more pronounced as the size of the portfolio increases and the value of δ increases with N, although this is not the case for the largest portfolio. The MGARCH models dominate the semi-parametric methods for all portfolio sizes. For $N = 5$ there is a gain in moving to MIDAS from either of the moving average approaches, and EWMA is found to be superior to the SMA. It is

[10] This level of risk aversion is considered to be an appropriate choice, as Ghysels, Santa-Clara and Valkanov (2005) have previously found the coefficient to be 2.6. Fleming et al. (2001, 2003) used coefficients of 1 and 10 to represent investors with relatively low and high risk aversion, respectively.

$N=5$							
	EQW	MIDAS	SMA	EWMA	CCC	cDCC	DECO
EQW	-	419.372 0.926	439.547 0.950	401.228 0.908	516.890 0.986	481.369 0.978	463.088 0.976
MIDAS		-	-55.242 0.250	-6.095 0.478	56.587 0.586	67.513 0.642	68.901 0.656
SMA			-	18.800 0.604	106.623 0.762	108.611 0.826	110.488 0.830
EWMA				-	60.508 0.568	74.314 0.614	76.591 0.622
CCC					-	8.359 0.540	15.223 0.770
cDCC						-	8.358 0.774
DECO							-

Table 3.8: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average value of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 5 assets.

$N=10$							
	EQW	MIDAS	SMA	EWMA	CCC	cDCC	DECO
EQW	-	1149.520 0.998	1198.317 0.998	1155.391 0.996	1428.740 1.000	1415.826 1.000	1430.802 1.000
MIDAS		-	-67.289 0.278	6.132 0.642	183.908 0.714	222.009 0.778	248.684 0.828
SMA			-	24.040 0.650	250.720 0.958	277.956 0.982	307.502 0.988
EWMA				-	177.468 0.696	217.345 0.754	243.764 0.798
CCC					-	27.549 0.816	62.597 0.940
cDCC						-	32.797 0.862
DECO							-

Table 3.9: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average value of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 10 assets.

worth noting that δ is relatively small when moving from EWMA to MIDAS and indeed the superiority of MIDAS is reversed for the moderate portfolio sizes $N = 10, 25$. For $N = 25$ there is a gain in switching from EWMA to the SMA, and this remains the case for the large portfolios of $N = 50, 100$.

As mentioned above, DECO outperforms all other methods by this measure across the various portfolios. On balance, the results presented here favour the assumption of equicorrelation especially for large portfolios. Despite the overall good performance of cDCC, the instability of portfolio weights it generates reduce the gains of using the cDCC to produce forecasts of the correlation matrix. This is most evident for the large portfolio sizes of 50 and 100 assets.

$N = 25$							
	EQW	MIDAS	SMA	EWMA	CCC	cDCC	DECO
EQW	-	293.490 0.900	381.746 0.972	290.702 0.896	431.637 1.000	438.389 1.000	466.123 1.000
MIDAS		-	20.426 0.592	-0.759 0.468	70.796 0.618	102.309 0.668	134.850 0.722
SMA			-	-48.196 0.316	42.332 0.590	68.851 0.672	103.006 0.758
EWMA				-	71.753 0.622	104.052 0.680	136.387 0.720
CCC					-	26.299 0.818	63.339 0.978
cDCC						-	35.079 0.852
DECO							-

Table 3.10: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average value of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 25 assets.

$N = 50$							
	EQW	MIDAS	SMA	EWMA	CCC	cDCC	DECO
EQW	-	19.603 0.556	169.587 0.860	26.432 0.570	191.703 0.950	214.943 0.936	267.253 0.984
MIDAS		-	94.342 0.862	11.364 0.702	126.623 0.850	170.558 0.952	231.770 0.982
SMA			-	-104.281 0.100	23.612 0.606	63.672 0.760	126.227 0.888
EWMA				-	114.750 0.832	159.290 0.932	220.427 0.980
CCC					-	39.379 0.884	105.481 0.984
cDCC						-	63.775 0.918
DECO							-

Table 3.11: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average value of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 50 assets.

3.6 Conclusion

This chapter presents an empirical study of the DECO model in comparison to other popular correlation forecasting techniques, all suitable for large dimensions, in the context of economic value. In particular, the question of whether complex specifications are necessary to produce superior forecasts of the correlation matrix is addressed. Out-of-sample forecasting performance is compared through the volatility of global minimum variance portfolio returns, portfolio stability and the explicit economic value of switching from one method to another.

$N = 100$							
	EQW	MIDAS	SMA	EWMA	CCC	cDCC	DECO
EQW	-	90.486 0.712	166.719 0.916	92.888 0.718	204.347 0.988	258.357 0.990	272.901 1.000
MIDAS		-	29.255 0.652	1.156 0.506	67.336 0.742	133.432 0.938	155.178 0.940
SMA			-	-53.617 0.230	26.105 0.638	89.182 0.884	112.965 0.932
EWMA				-	66.041 0.740	132.340 0.956	153.966 0.962
CCC					-	62.016 0.982	88.035 0.998
cDCC						-	23.574 0.746
DECO							-

Table 3.12: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average value of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 100 assets.

DECO provides the lowest variance and over the entire sample studied is included in the Model Confidence Set (MCS) for all portfolio sizes. It also delivers the most stable portfolio in terms of asset weights of the techniques compared across the various portfolio sizes. The incremental economic value of moving from another method to equicorrelation is positive. The out-of-sample period is broken into subsamples of high and low volatility to further evaluate the forecasts. DECO is found to perform particularly well during the crisis period across the various portfolios and CCC performs well during the second period of market calm (post-GFC). These results indicate that complex specifications such as the MGARCH framework produce superior forecasts in comparison to simple moving average style models. However, more basic versions of the MGARCH family are adequate in certain scenarios.

The differences between model forecasting performance during subsamples of high and low volatility are worth further investigation. One of the interesting conclusions to emerge from the research reported in this chapter is that correlations are stable when volatility is low. This observation suggests a potential avenue for further consideration of equicorrelation models, namely the introduction of volatility as a determinant of equicorrelation.

Chapter 4

Volatility Dependent Dynamic Equicorrelation

4.1 Introduction and Motivation

The previous chapter highlighted the potential advantages of the equicorrelation framework in the context of managing large portfolios. Developing methods appropriate for forecasting the correlation matrices of these large systems is an important and relevant problem, with a host of financial applications. The focus of this chapter is to investigate a link between volatility and correlations by conditioning the equicorrelation process on volatility. The Volatility Dependent Dynamic Conditional Correlation class of model of Bauwens and Otranto (2013)[1] is adapted to the equicorrelation context. The usefulness of assuming equicorrelation for the purposes of forecasting the correlation matrix, especially in times of market turbulence, has been shown in Engle and Kelly (2012) and confirmed in Chapter 3. The comparative ease with which this class of model can be estimated and used to generate forecasts of large correlation matrices provides motivation for use of the equicorrelation framework.

Two empirical applications of the correlation forecasting models are presented in this chapter. The first investigates the relationship between volatility and correlations in the context of the U.S. equity market, examining volatility as a determinant of correlations

[1] A paper of the same name has recently been published by Luc Bauwens and Edoardo Otranto. The published version appears in the *Journal of Business & Economic Statistics* (2016), Volume 34, Issue 2, pp. 254–268.

in a single national market. Various equity portfolio sizes are used here, ranging from 5 assets through to 100. Secondly, the methodology is applied to a set of European market indices with the intention of linking this chapter with the international work of others, discussed in Section 2.5.4. Additionally, this chapter studies whether the models used here consistently produce superior forecasts in both national and international contexts. The Volatility Index (VIX) is used in both examples to represent volatility of the U.S. equities market, often used as a proxy for global equity markets.

The correlation forecasting methods are compared using a portfolio allocation problem, similar to that outlined in Chapter 3. Evaluation of these methods center on the formation of global minimum variance portfolios and use of the Model Confidence Set to compare the various forecasting methods, as well as measuring the relative economic value of the models. The empirical applications presented differ from Bauwens and Otranto (2013), as the focus here is tractability in large dimensional problems and specifically correlation forecasting. All correlation forecasting models compared in this chapter are applicable in large dimensions, unlike Bauwens and Otranto (2013) where their choice of models limits the portfolio size in the empirical application.

In the context of the U.S. equity market, the equicorrelation family of models perform well against the cDCC-based methods consistently across various portfolio sizes. For large portfolios a simple specification such as constant conditional correlation seems sufficient, particularly during periods of market calm. Based on the evidence presented in this chapter, there is a strong case for the use of an equicorrelation structure rather than a cDCC-based one. This is certainly the case during periods of market turbulence. These results are consistent with the findings outlined in Chapter 3. The comparison of the Volatility Dependent Dynamic Conditional Correlation (VDCC) framework and the Volatility Dependent Dynamic Equicorrelation (VDECO) models presented here strengthens this evidence. For the VDCC models, there appears to be evidence in favour of conditioning the correlation structure directly on volatility.

The correlation forecasting models are also applied to a set of 14 European indices. In contrast to the U.S. equities example, the equicorrelated models perform poorly against the cDCC-based methods across all metrics used in this chapter. Perhaps this is due to the construction of indices as opposed to individual equities, as DECO's advantage is one of information pooling.[2] Use of indices as opposed to individual equities allows the cDCC models to exploit pooled information, thus eroding the advantage of an equicorrelation framework. The cDCC model's ability to track the correlation dynamics of the portfolio is subsequently more effective in this context. Regarding the VDCC models, there is a definite advantage in extending the standard cDCC framework to condition on volatility although which is the best specification to use varies over the sample. In both the U.S. and European applications, there appears to be no statistically significant difference between the standard equicorrelation model and the Volatility Dependent class although in general a volatility dependent structure leads to lower portfolio variances.

4.2 Methodology

Much of the methodology used here has been discussed in detail in preceding chapters, see in particular Section 3.2 of Chapter 3. As before, estimation of the conditional covariance matrix equation 3.1 is performed in two stages. In the case of the first stage of the estimation, $\mathbf{D}_t$ in equation 3.1, the GJR–GARCH model of Glosten et al. (1993) is used. Volatility persistence in equity returns and the asymmetry common in the volatility of equity returns motivates the use of these models in the univariate context and this method is standard procedure in the MGARCH literature. The following sections detail modelling the conditional correlation matrix, $\mathbf{R}_t$ in equation 3.1, performed as the second stage of estimation. For comparison purposes the Constant Conditional Correlation (CCC) model of Bollerslev (1990) is estimated, where the conditional correlations are assumed to be constant over time.

4.2.1 Volatility Dependent Dynamic Equicorrelation

This section outlines the volatility dependent equicorrelated (VDECO) models, extending the cDCC-based Volatility Dependent Dynamic Conditional Correlation (VDCC) models of Bauwens and Otranto (2013) to the equicorrelation framework. The VDCC methodology has been outlined in Section 2.5.4 of Chapter 2 and the models relevant here are provided in equations 2.55 to 2.59.

[2]Engle and Kelly (2012) explain this is due to DECO's use of the history of all pairs of assets for each forecast, rather than cDCC using the history of the particular pair of assets under consideration.

Recall from Chapter 2 that the DECO model of Engle and Kelly (2012) specifies the conditional correlation matrix, $\mathbf{R}_t$, as

$$\mathbf{R}_t = (1 - \rho_t)\mathbf{I}_N + \rho_t \mathbf{1}_N \ . \tag{4.1}$$

Here ρ_t is the scalar equicorrelation measure, $\mathbf{I}_N$ the N-dimensional identity matrix and $\mathbf{1}_N$ a $N \times N$ matrix of ones. The equicorrelation ρ_t is formed by averaging the pairwise cDCC pseudo-correlations given in equation 3.2,

$$\rho_t^{cDCC} = \frac{1}{N(N-1)}\left(\mathbf{1}_N' \mathbf{R}_t^{cDCC} \mathbf{1}_N - N\right) = \frac{2}{N(N-1)} \sum_{n>m} \frac{q_{n,m,t}}{\sqrt{q_{n,n,t} q_{m,m,t}}} \ , \tag{4.2}$$

where $q_{n,m,t}$ is the n,mth element of the pseudo-correlation matrix, $\mathbf{Q}_t$.

From equation 4.2 it is clear that the volatility dependent set of models follow the same logic. The VDCC pseudo-correlations specified in equations 2.55, 2.56, 2.57 and 2.59 replace $\mathbf{R}_t^{cDCC}$ in equation 4.2. For example, the DEC-AVE model is formed using the pairwise pseudo-correlations given by $\mathbf{Q}_t^{DCC-AVE}$ in equation 2.55,

$$\rho_t^{DCC-AVE} = \frac{1}{N(N-1)}\left(\mathbf{1}_N' \mathbf{R}_t^{DCC-AVE} \mathbf{1}_N - N\right) = \frac{2}{N(N-1)} \sum_{n>m} \frac{q_{n,m,t}}{\sqrt{q_{n,n,t} q_{m,m,t}}} \ . \tag{4.3}$$

Similar to equation 4.2 above, $q_{n,m,t}$ is the n,mth element of the pseudo-correlation matrix, $\mathbf{Q}_t^{DCC-AVE}$.

In keeping with the terminology above, the VDECO class of models are referred to as DEC-AVE (in equation 4.3), DEC-ARE, DEC-TVV and DEC-TVR respectively. It should be noted that Bauwens and Otranto (2013) use (VIX/100) in their estimation of models 2.55 to 2.59. Here, preliminary experiments found log(VIX) as v_{t-1} to be effective in the additive -AVE and -ARE models.

A two state Markov switching model, in equation 2.28, is used for the volatility regimes of the VIX. Two states are found to be sufficient over the time period of 1996 to 2012. This is consistent with Sarwar (2014), who studies a similar period and also identifies two distinct regimes. The raw VIX is used to model regime switching.

To circumvent the intensive estimation associated with the cDCC models, composite likelihood (CL) estimation (Pakel et al., 2014) is used to estimate the VDCC parameters.

The CL is a sum of quasi-likelihoods obtained by breaking the portfolio into smaller subsets, in this case unique pairs of assets form these subsets. An outline of CL estimation in the context of MGARCH estimation is provided in Chapter 2 and the specific methodology applied here is presented in Chapter 3.

Evaluation of the correlation forecasts for each method follows the practical approach presented in Section 3.2.2 of Chapter 3. The volatility of global minimum variance (GMV) portfolios is compared in order to evaluate forecasting performance of the competing models and statistical significance of any differences examined using the Model Confidence Set (MCS) of Hansen et al. (2011). The economic value of each correlation forecasting method is then quantified by examining the incremental gain of switching from a particular model to another using the methodology of Fleming et al. (2003). This evaluation framework is carried through both the domestic and international examples.

4.3 The Domestic Context: U.S. Equities

This section outlines the results of the first empirical study, based on the U.S. equities market. The portfolios used contain a selection of S&P 500 members that are continuously traded over the period 3 January 1996 to 31 December 2012. The full dataset contains 100 stocks and 4269 observations, with various portfolio sizes of $N = 5, 10, 25, 50, 100$ stocks chosen at random from the full list. All GICS sectors are represented across the dataset and the full list of stocks including their ticker code, company name and sector is provided in Appendix B. Log returns are calculated using $r_{n,t} = \log p_{n,t} - \log p_{n,t-1}$, where $p_{n,t}$ denotes the daily closing price of asset n at time t, adjusted for stock splits and dividends. The VIX index is available from the Chicago Board Options Exchange and is constructed using out-of-the-money put and call options that have maturities of 22 trading days. It is a measure of the implied volatility of S&P 500 index over the next 22 trading days and further details are included in Section 2.3.3 of Chapter 2. Technical details of the VIX can be found through the Chicago Board Options Exchange (see CBOE, 2014).

Figure 4.1 shows the VIX and daily returns of the S&P 500 index respectively. This figure emphasises the relationship between the index and the VIX, as daily returns of the index vary considerably during 2008. This period of extreme volatility in the second half of the dataset is highlighted by the plot of the VIX, with the highest point 80.86 corresponding to 20 November 2008 and the global financial crisis (GFC). These periods of relative high and low volatility over the sample are of interest in the application of the volatility dependent correlation structures proposed in this chapter. Subsequently, the analysis of the forecasting methods includes their comparative performance over subsamples of differing levels of volatility.

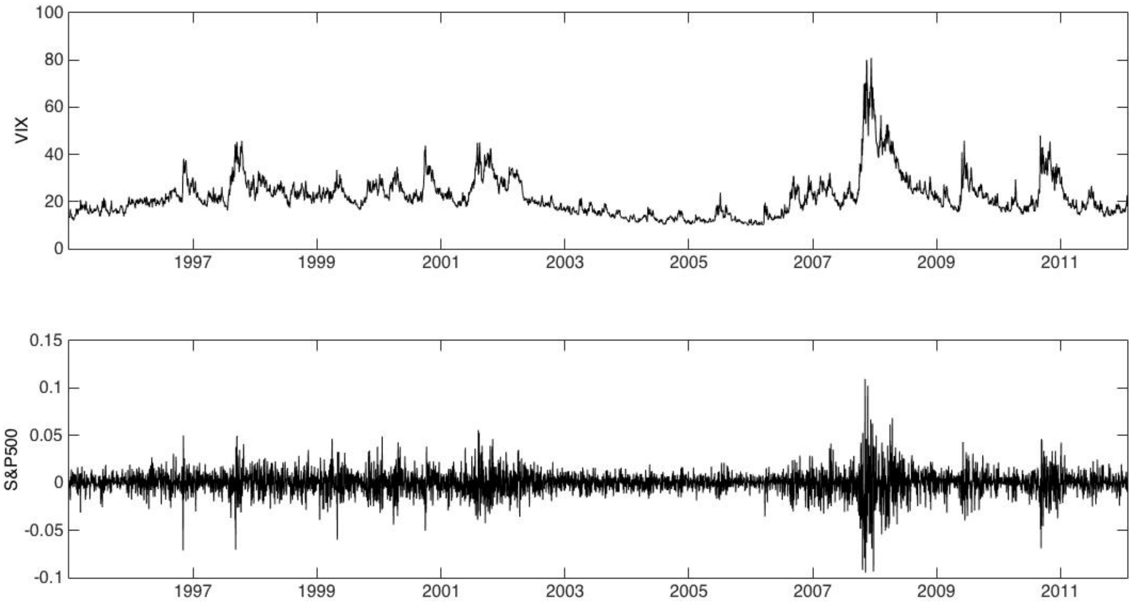

Figure 4.1: Level of the VIX (top) and daily returns of the S&P 500 index (bottom), entire period spans 3 January 1996 to 31 December 2012.

4.3.1 Univariate Model Estimation

As was the case in Chapter 3, preliminary experiments found nine stocks to have insignificant asymmetry coefficients. Their volatility processes are estimated using GARCH, leaving 91 stocks with univariate volatility processes estimated using GJR–GARCH. For the VIX, a two state Markov Switching (MS) model is estimated to obtain the expected probability of the high volatility regime, $\mathrm{E}_{t-1}(\zeta_{t-1})$, at time $t-1$. Further detail regarding the MS-AR(2) model is contained in Section 4.2.1.

Figure 4.2 illustrates the effectiveness of using the MS-AR(2) model to estimate the regimes of the volatility measure. The VIX is shown along with the updated one-step-ahead filtered probability of being in the high volatility state. Unsurprisingly, the model predicts the high volatility state as having a higher probability more frequently during the GFC in the middle section of the out-of-sample period. The later periods of relatively high volatility correspond to the worsening European debt crisis.

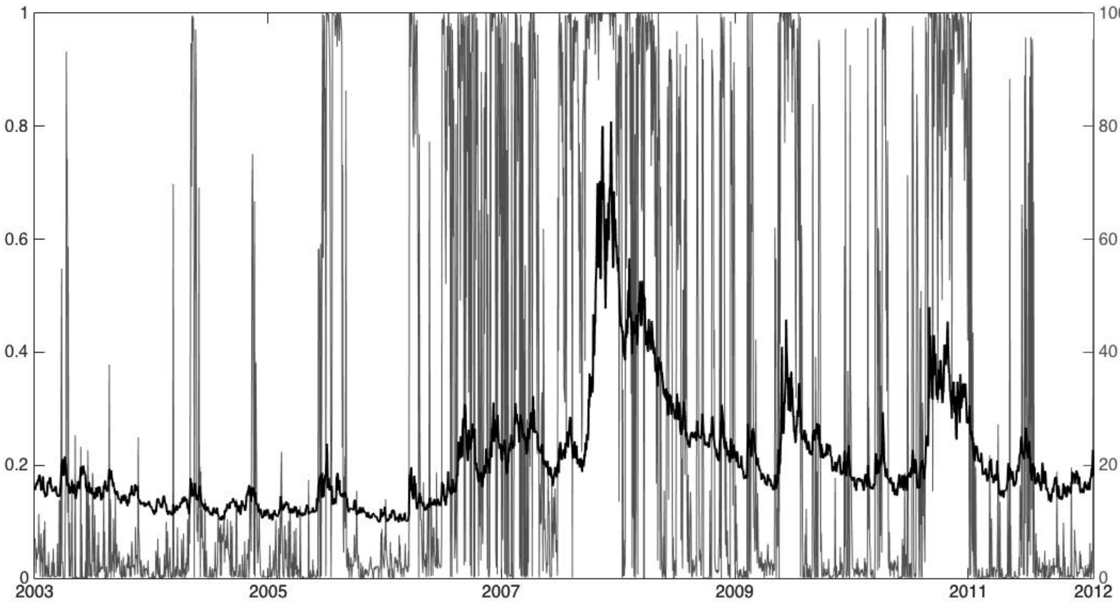

Figure 4.2: Out-of-sample filtered probabilities of high volatility regime of VIX estimated by a two state MS model (left axis). VIX over the out-of-sample period (right axis). In sample period is 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

4.3.2 Full Sample Results

Table 4.1 contains full sample parameter estimates for each of the correlation models across the various portfolio sizes. During preliminary estimation experiments, the time varying coefficient b_t in equation 2.59 was found to be constant. Subsequently the parameters b_1, $\theta_{b,0}$ and $\theta_{b,1}$ for the DCC-TVV, DCC-TVR, DEC-TVV and DEC-TVR models are suppressed in the results reported here and throughout this chapter.[3]

In terms of the VDCC models, the additive parameter g (governing the volatility term) is close to 0. This confirms similar results found by Bauwens and Otranto (2013). DCC-TVV and DCC-TVR provide similar parameter estimates for the smallest portfolio sizes of $N = 5$ and 10, however the distribution of a_0 and a_1 is different for $N = 25$, 50 and 100.

[3]Bauwens and Otranto (2013) also find b_t to be constant and suppress the relevant parameters accordingly.

In the context of the large portfolio sizes, the time varying component of the VDCC is statistically significant. Interestingly, it is the additive effect that appears to be more useful for the VDCC in the forecasting examples. The relevance of the additive volatility term in a practical application of the model suggests re-estimation of this parameter is potentially important. It would be reasonable to suppose differing market volatility conditions over time drives this effect. See Sections 4.3.3 and 4.4.3 for detailed discussion of economic significance.

In the case of the VDECO family, the parameter g is close to 0. However, for the moderate and large portfolios, $N = 50$ and 100, there is a significant volatility effect for the level of the VIX (DEC-AVE). This is not the case for the corresponding regime model DEC-ARE. As Bauwens and Otranto (2013) limit their empirical study to 30 stocks, these results pertaining to the VDECO family's behaviour in the context of large dimensions are of particular interest. For the time varying volatility dependent DECO models, the full sample parameter estimates of the DEC-TVV and DEC-TVR are very similar across the various portfolio sizes. In general, the time varying a_t is much larger for the VDECO class than the VDCC models. In a forecasting sense, this appears important[4] and further discussion on the economic value of these results is contained in Section 4.3.3.

It is worth noting that the addition of the time varying coefficient a_t to the original DECO model seems to result in a decrease in the value of the parameter b and increase in a_t, compared to the constant parameter a. An implication of this is that the distribution between the present and past information making up the measure of persistence in the correlations changes. The 10-asset portfolio appears to be anomalous in this sense. Similar differences appear in the distribution between the parameters α and β when comparing the original cDCC and DECO models. That is, the cDCC estimates higher β parameters and lower α values than the equicorrelated model. This difference is irrespective of portfolio size and was documented in Engle and Kelly (2012).

The log-likelihood values for each of the models and information based ranking criteria values, specifically AIC and BIC, are contained in Appendix B (Tables B.1 and B.2).[5] All log-likelihood values are generated using the original cDCC log-likelihood equation, for

[4]Particularly in the context of U.S. equities.

[5]Several additional tables relevant to the analysis in this chapter are contained in Appendix B and referred to where necessary to avoid dilution of the main text.

U.S. Equities: Full sample parameter estimates

N	Model	a	a_0	a_1	b	g	$\theta_{a,0}$	$\theta_{a,1}$	Model	a	a_0	a_1	b	g	$\theta_{a,0}$	$\theta_{a,1}$
5	cDCC	0.0118 (0.0041)			0.9773 (0.0115)				DEC	0.0417 (0.0136)			0.9443 (0.0264)			
	DCC-AVE	0.0115 (0.0037)			0.9794 (0.0093)	0.0001 (0.0001)			DEC-AVE	0.0414 (0.0127)			0.9474 (0.0195)	0.0003 (0.0003)		
	DCC-TVV		0.0015 (0.0070)	0.0202 (0.0216)	0.9792 (0.0094)		−0.0000 (0.0011)	0.0013 (0.0031)	DEC-TVV		0.0558 (0.0251)	0.0479 (0.0088)	0.8724 (0.0695)		0.0004 (0.0009)	0.0014 (0.0269)
	DCC-ARE	0.0119 (0.0035)			0.9790 (0.0085)	−0.0005 (0.0006)			DEC-ARE	0.0415 (0.0133)			0.9447 (0.0269)	0.0002 (0.0023)		
	DCC-TVR		0.0013 (0.0133)	0.0207 (0.0350)	0.9792 (0.0104)		−0.0001 (0.0159)	0.0012 (0.0005)	DEC-TVR		0.0559 (0.0270)	0.0479 (0.0037)	0.8724 (0.0657)		0.0004 (0.0003)	0.0003 (0.0024)
10	cDCC	0.0024 (0.0003)			0.9948 (0.0012)				DEC	0.0449 (0.0106)			0.9274 (0.0251)			
	DCC-AVE	0.0066 (0.0017)			0.9854 (0.0055)	0.0000 (0.0001)			DEC-AVE	0.0447 (0.0105)			0.9271 (0.0255)	−0.0001 (0.0004)		
	DCC-TVV		−0.0063 (0.0002)	0.0164 (0.0001)	0.9959 (0.0007)		−0.0001 (0.0009)	0.0010 (0.0003)	DEC-TVV		0.0279 (0.0064)	0.0339 (0.0117)	0.9276 (0.0226)		−0.0001 (0.0044)	−0.0015 (0.0053)
	DCC-ARE	0.0068 (0.0012)			0.9857 (0.0034)	−0.0011 (0.0004)			DEC-ARE	0.0449 (0.0139)			0.9274 (0.0270)	0.0000 (0.0033)		
	DCC-TVR		−0.0063 (0.0001)	0.0168 (0.0018)	0.9956 (0.0008)		−0.0001 (0.0005)	0.0010 (0.0007)	DEC-TVR		0.0276 (0.0115)	0.0338 (0.0016)	0.9282 (0.0206)		−0.0001 (0.0023)	−0.0015 (0.0008)
25	cDCC	0.0024 (0.0002)			0.9938 (0.0009)				DEC	0.0473 (0.0115)			0.9224 (0.0188)			
	DCC-AVE	0.0168 (0.0240)			0.9663 (0.0518)	0.0000 (0.0002)			DEC-AVE	0.0553 (0.0157)			0.9385 (0.0151)	0.0007 (0.0001)		
	DCC-TVV		0.0057 (0.0143)	0.0223 (0.0007)	0.9661 (0.0299)		−0.0001 (0.0042)	−0.0001 (0.0015)	DEC-TVV		0.0528 (0.0231)	0.0464 (0.0026)	0.8681 (0.0470)		0.0004 (0.0031)	0.0011 (0.0034)
	DCC-ARE	0.0061 (0.0016)			0.9863 (0.0041)	−0.0020 (0.0008)			DEC-ARE	0.0473 (0.0153)			0.9225 (0.0240)	0.0000 (0.0029)		
	DCC-TVR		0.0101 (0.0055)	0.0135 (0.0047)	0.9661 (0.0167)		0.0000 (0.1184)	0.0002 (0.0686)	DEC-TVR		0.0528 (0.0360)	0.0464 (0.0784)	0.8681 (0.0126)		0.0004 (0.1190)	0.0001 (0.0197)
50	cDCC	0.0008 (0.0001)			0.9961 (0.0002)				DEC	0.0474 (0.0098)			0.9204 (0.0171)			
	DCC-AVE	0.0158 (0.0022)			0.9703 (0.0043)	0.0000 (0.0001)			DEC-AVE	0.0590 (0.0140)			0.9374 (0.0132)	0.0006 (0.0001)		
	DCC-TVV		0.0050 (0.0015)	0.0220 (0.0002)	0.9697 (0.0026)		−0.0002 (0.0186)	−0.0002 (0.0064)	DEC-TVV		0.0517 (0.0094)	0.0459 (0.0190)	0.8656 (0.0404)		0.0004 (0.0009)	0.0011 (0.0020)
	DCC-ARE	0.0017 (0.0001)			0.9932 (0.0006)	−0.0006 (0.0002)			DEC-ARE	0.0474 (0.0177)			0.9204 (0.0307)	0.0000 (0.0040)		
	DCC-TVR		0.0094 (0.0010)	0.0132 (0.0002)	0.9697 (0.0020)		0.0003 (0.0000)	0.0002 (0.0006)	DEC-TVR		0.0518 (0.0356)	0.0459 (0.0430)	0.8656 (0.0158)		0.0004 (0.1962)	0.0002 (0.0105)
100	cDCC	0.0034 (0.0002)			0.8441 (0.0172)				DEC	0.0532 (0.0089)			0.9057 (0.0144)			
	DCC-AVE	0.0164 (0.0008)			0.9662 (0.0018)	−0.0000 (0.0001)			DEC-AVE	0.0804 (0.0166)			0.8574 (0.0317)	0.0006 (0.0005)		
	DCC-TVV		0.0052 (0.0003)	0.0221 (0.0001)	0.9667 (0.0001)		−0.0001 (0.0002)	−0.0001 (0.0006)	DEC-TVV		0.0513 (0.0050)	0.0457 (0.0128)	0.8648 (0.0310)		0.0004 (0.0021)	0.0011 (0.0039)
	DCC-ARE	0.0036 (0.0002)			0.8316 (0.0153)	−0.0064 (0.0021)			DEC-ARE	0.0533 (0.0333)			0.9056 (0.0598)	0.0000 (0.0091)		
	DCC-TVR		0.0096 (0.0003)	0.0133 (0.0001)	0.9666 (0.0006)		0.0000 (0.0002)	0.0002 (0.0005)	DEC-TVR		0.0514 (0.0182)	0.0457 (0.0094)	0.8648 (0.0255)		0.0004 (0.0043)	0.0001 (0.0006)

Table 4.1: Parameter estimates of models for period 3 January 1996 to 31 December 2012 for each portfolio. Robust standard errors in parentheses.

all models, to ensure comparable values. The VDCC and VDECO are broadly similar in terms of log-likelihood values, with no estimation problems evident. Using this criteria it is difficult to draw conclusions regarding which models exhibit better fit over the sample.

4.3.3 Out of Sample Forecasts

An initial in-sample period of 2000 observations is used, giving an out-of-sample period of $T = 2269$. The forecasting horizon is one day and correlation parameters are re-estimated over an expanding window every 5 observations (the equivalent of one trading week). To illustrate the differences between the VDCC and VDECO families of models, Figures 4.3 through 4.5 show the average daily return of $N = 5$, 25 and 100 portfolios, along with the one-step-ahead forecasts of equicorrelation (bottom) from the DECO model and average correlation forecasts of the cDCC model (top). These portfolio sizes are chosen to illustrate small, moderate and large dimensions; $N = 10,\ 50$ are contained in Appendix B (Figures B.1 and B.2). In the case of the VDCC family, elements of the correlation matrix are averaged to provide an estimate of equicorrelation, $\bar{\rho}_t$, for comparison purposes. Across all portfolio sizes the cDCC produces generally smoother average correlation forecasts than the DECO model. Also clear from the figures is the seemingly higher level of correlations seen post-GFC, with the forecasted correlations rising more sharply than during the GFC itself. This period of higher market volatility corresponds to the European sovereign debt crisis and speaks to increasing global integration (see Christoffersen et al., 2014) as possible reasoning behind the difference between the pre- and post-GFC sub-periods. To further examine the differences in correlation forecasts generated by the VDCC and VDECO models, Tables B.3 through B.6 (included in Appendix B) provide the average equicorrelation, $\bar{\rho}$, and standard deviation of the equicorrelation forecasts. Generally, the VDCC models provide lower standard deviations than the VDECO family and the average level of equicorrelation is similar. The higher variation in the VDECO correlations appears due to the decrease in parameter b and corresponding increase in the time varying parameter a_t, discussed in Section 4.3.2. The results are consistent across all time periods, although the average level of correlations increases for the high volatility period as expected. This level stays high for the post-GFC period of market calm, in

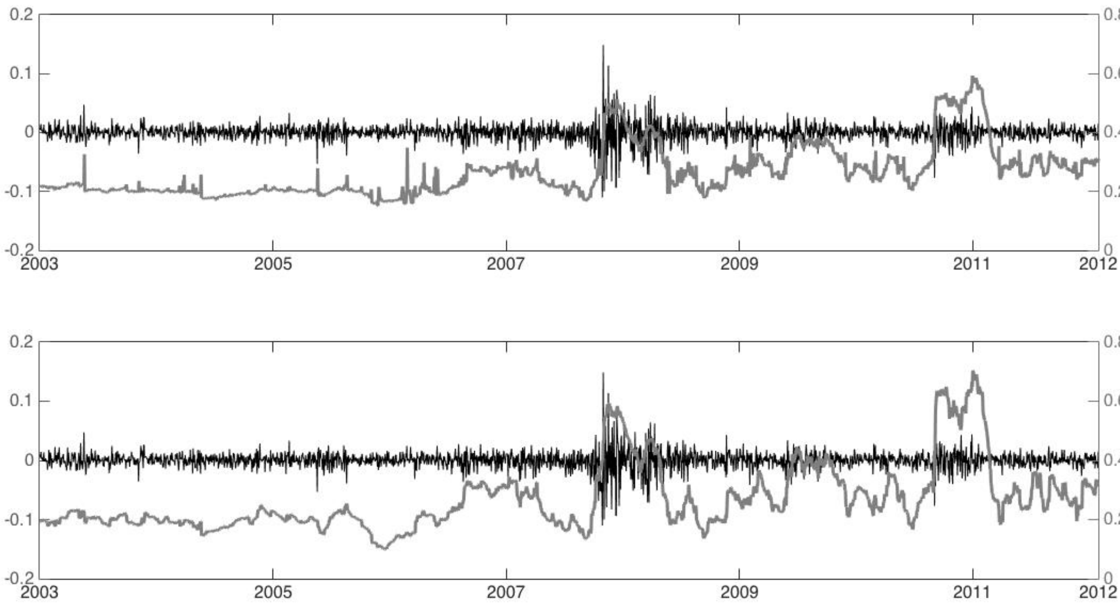

Figure 4.3: Average daily return of portfolio of 5 U.S. equities for out-of-sample period (left axis). One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC model (top, right axis). One-step-ahead equicorrelation forecasts, ρ_t (bottom, right axis). Entire period spans 3 January 1996 to 31 December 2012.

comparison to the low level seen in the first low volatility sub-period. Such differences indicate market volatility has not yet reverted to levels seen pre-GFC.

Results presented in Table 4.2 are the out-of-sample standard deviations of the GMV portfolio described in Section 3.2.2 of Chapter 3, across the various portfolios sizes and models. The CCC performs well in terms of providing the lowest standard deviation for the large portfolios of $N = 50$ and 100 however provides higher volatility for the small and moderate portfolio sizes. The previous chapter underscored the effectiveness of CCC, particularly under calm market conditions and in large dimensional systems. This is consistent with the findings here. VDCC appears to perform poorly overall, providing relatively higher standard deviations than VDECO and CCC as portfolio size increases. Within the VDCC family of models, DCC-AVE[6] provides the lowest standard deviations for all portfolios except $N = 25$ where cDCC provides the lower portfolio volatility. The direct approach of an additive volatility term is useful in the VDCC context. The original DECO model, without volatility dependence, provides higher volatilities in general than the VDECO extension. In contrast to the VDCC family, allowing the volatility dependence

[6]Note all VDCC models are based on the cDCC of Aielli (2013), however the lower case 'c' is omitted from the relevant acronyms for consistency with Bauwens and Otranto (2013).

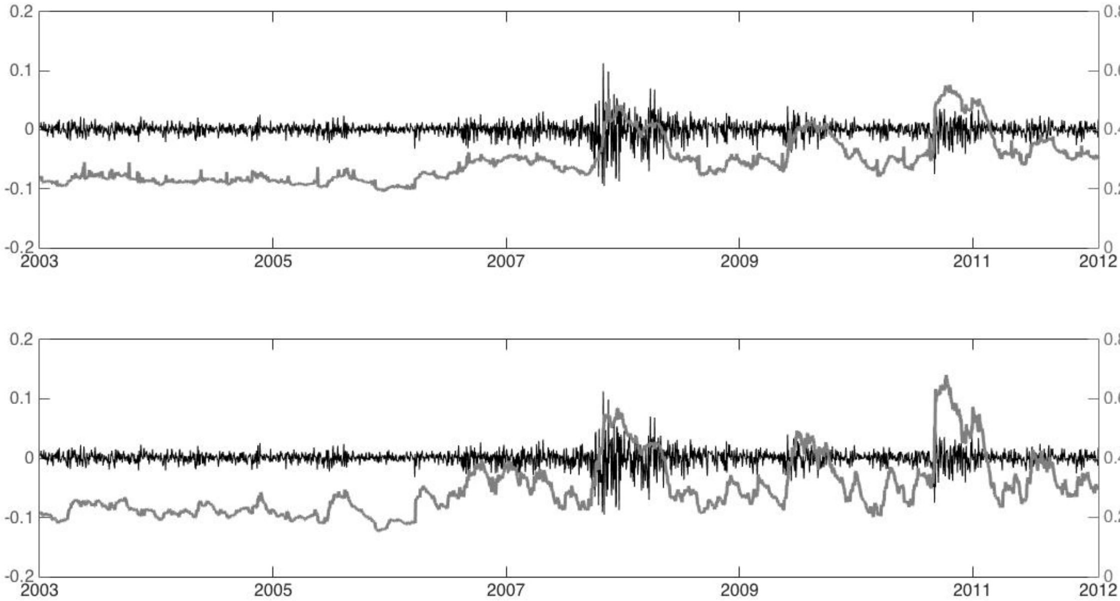

Figure 4.4: Average daily return of portfolio of 25 U.S. equities for out-of-sample period (left axis). One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC model (top, right axis). One-step-ahead equicorrelation forecasts, ρ_t (bottom, right axis). Entire period spans 3 January 1996 to 31 December 2012.

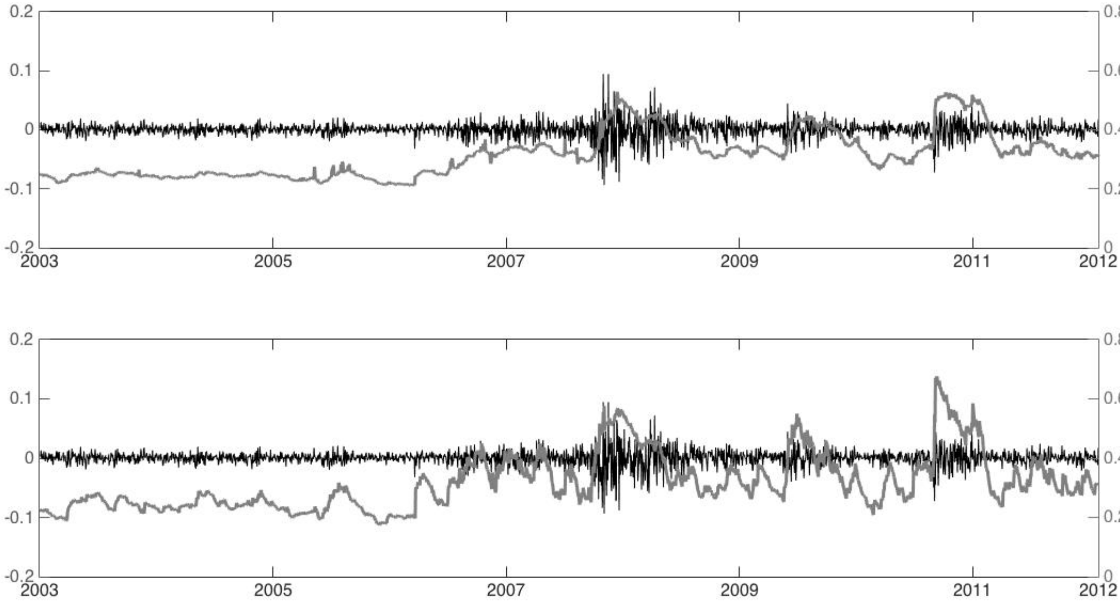

Figure 4.5: Average daily return of portfolio of 100 U.S. equities for out-of-sample period (left axis). One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC model (top, right axis). One-step-ahead equicorrelation forecasts, ρ_t (bottom, right axis). Entire period spans 3 January 1996 to 31 December 2012.

to indirectly enter the correlation structure via the time varying parameter a_t leads to lower standard deviations than the direct additive term. In general, allowing the correlation process to be dependent on a volatility component appears a more effective extension in the context of the cDCC-based models. The linkage between correlations and volatility is less beneficial in the equicorrelated scenario. Attention now turns to whether the differences seen in the GMV portfolio standard deviations (reported in Table 4.2) are statistically significant.

The MCS is used to statistically distinguish between the forecast performance of the models based on the volatilities of past GMV portfolio returns. These results are presented in Table 4.3. The MCS contains the best model(s) with a level of confidence of 95% (see Section 3.2.2 of Chapter 3 for further details). Overall, the VDECO family is the most consistent across the various portfolio sizes, with the DCC, DECO, DEC-TVV and DEC-TVR models included in each MCS for all portfolios. For the smallest portfolios, all models are included in the MCS and unsurprisingly the better models within the MCS closely follow the trends of Table 4.2. For the 25 asset portfolio, CCC, DCC-AVE and DEC-AVE are excluded from the MCS. CCC is included with a p-value of 1 for both $N = 50$ and 100. This means it would be the last model excluded from the set. While conditioning on volatility may reduce portfolio volatility in general, differences in performance are not statistically significant in this empirical example. Indeed, the only statistically significant differences are found in the moderately sized portfolios ($N = 25$, 50). The implication of this is that there is a difference in the type of correlation forecast needed for small versus large portfolios, emphasised by the relative success of the CCC in the largest cases. The cDCC model also performs well over the various sizes of N. Despite the few statistically significant differences found, assuming equicorrelation is the most consistently successful method regardless of portfolio size (leading to smaller portfolio volatilities than cDCC).

From a risk management viewpoint, it is arguably more important to forecast correlations accurately during periods of crisis. For this reason the out-of-sample period is split into subsamples of relatively high and low volatility to gain a deeper understanding of forecasting performance. The out-of-sample standard deviations of each of the GMV portfolios over the sub-periods are contained in Table 4.4. As was the case with the entire sample, the CCC model provides the lowest standard deviations in the largest case and

U.S. Equities: GMV portfolio forecasts

N	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
5	18.1505	17.4056	17.2667	17.3608	17.4562	17.3533	17.0610	17.0465	17.0558	17.0804	17.1870
10	17.6654	17.0159	16.7828	17.0093	17.0158	17.0216	17.0018	16.9952	16.9023	17.0347	16.9320
25	13.3219	13.2224	13.2283	13.2478	13.2413	13.2461	12.9380	13.2594	12.8084	12.9898	12.8152
50	12.6511	12.9458	12.8973	12.9701	12.9504	12.9480	12.8017	13.0432	12.7722	12.9171	12.7653
100	11.9346	12.6796	12.5819	12.7005	12.6667	12.6849	12.5128	12.5993	12.4906	12.5770	12.5401

Table 4.2: Annualised percentage volatility of out-of-sample minimum variance portfolio returns for each volatility timing strategy. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

U.S. Equities: Model Confidence Set (MCS)

N	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
5	0.3200*	0.3390*	0.3390*	0.3390*	0.3390*	0.4900*	0.8300*	1.0000*	0.8560*	0.4900*	0.4900*
10	0.1550*	0.5960*	1.0000*	0.5960*	0.5120*	0.4830*	0.5120*	0.5120*	0.5960*	0.4410*	0.5960*
25	0.0000	0.0510*	0.0000	0.0510*	0.0510*	0.0510*	0.0670*	0.0000	1.0000*	0.0510*	0.1580*
50	1.0000*	0.1180*	0.1180*	0.0020	0.0020	0.0020	0.1600*	0.0000	0.8700*	0.0020	0.8700*
100	1.0000*	0.6960*	0.7970*	0.7590*	0.7970*	0.6960*	0.7970*	0.7970*	0.7970*	0.7970*	0.9940*

*Table 4.3: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.*

U.S. Equities: GMV portfolio forecasts, sub-periods

Period	N	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
Dec 2003 to Feb 2007 (Low 1)												
2001:2806	5	12.8808	12.8706	12.9619	12.8737	12.8861	12.8336	12.8630	12.8492	12.8647	12.8790	12.8424
	10	11.2450	11.1611	11.2394	11.1578	11.1539	11.1577	11.2466	11.2485	11.2075	11.2780	11.2074
	25	9.2577	9.2820	9.5796	9.2865	9.2890	9.2866	9.3675	9.8346	9.3448	9.3951	9.3474
	50	8.6187	8.7799	8.9762	8.7847	8.7458	8.7963	9.0658	9.4581	9.0856	9.2214	9.0796
	100	7.9048	8.3081	8.4616	8.3315	8.2895	8.3327	9.0026	9.1577	9.0063	9.1204	9.0689
Mar 2007 to Dec 2011 (High)												
2807:4019	5	21.9573	20.7996	20.5481	20.7281	20.8725	20.7334	20.2478	20.2266	20.2616	20.2694	20.4759
	10	21.8658	20.9129	20.5555	20.9039	20.9146	20.9225	20.8610	20.8572	20.7232	20.9000	20.7752
	25	16.1738	15.9881	15.8813	16.0257	16.0145	16.0231	15.5117	15.7839	15.3269	15.5817	15.3356
	50	15.3797	15.7395	15.5946	15.7753	15.7604	15.7372	15.2283	15.3897	15.1850	15.3483	15.1695
	100	14.5829	15.5036	15.3011	15.5273	15.4915	15.5028	14.8683	14.9372	14.8338	14.9220	14.8799
Dec 2011 to Dec 2012 (Low 2)												
4020:4268	5	10.6091	10.6487	10.6600	10.6499	10.6521	10.6501	10.8074	10.8526	10.6118	10.8302	10.6154
	10	10.2235	10.2038	9.9573	10.2047	10.2088	10.2047	10.2051	10.1354	10.2067	10.2052	10.1405
	25	8.0903	8.3138	8.3108	8.3125	8.3134	8.3125	8.3237	8.6907	8.2468	8.3217	8.2547
	50	8.1174	8.4209	8.3892	8.4203	8.4110	8.4199	9.8867	10.3457	9.8048	9.8861	9.8564
	100	7.7359	8.4066	8.3775	8.4068	8.3989	8.4067	9.3610	9.3971	9.3458	9.3606	9.3969

Table 4.4: Annualised percentage volatility of out-of-sample minimum variance portfolio returns for each volatility timing strategy, split into 'high' and 'low' volatility. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

U.S. Equities: MCS, sub-periods

Period	N	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
Dec 2003 to Feb 2007 (Low 1)												
2001:2806	5	0.1530*	0.1530*	0.1130*	0.1130*	0.1130*	1.0000*	0.1530*	0.2770*	0.1530*	0.1130*	0.9470*
	10	0.1750*	0.9970*	0.9900*	0.9970*	1.0000*	0.9970*	0.1750*	0.1750*	0.9970*	0.1750*	0.9970*
	25	1.0000*	0.8870*	0.0410	0.8870*	0.5080*	0.5080*	0.5080*	0.0010	0.5080*	0.5080*	0.5080*
	50	1.0000*	0.0790*	0.0280	0.0790*	0.0790*	0.0580*	0.0280	0.0130	0.0280	0.0280	0.0280
	100	0.9450*	0.7050*	1.0000*	0.9440*	0.9440*	0.8610*	0.9570*	0.9570*	0.9450*	0.9440*	0.9570*
Mar 2007 to Dec 2011 (High)												
2807:4019	5	0.2490*	0.2810*	0.2810*	0.2810*	0.2810*	0.2810*	0.8440*	1.0000*	0.8440*	0.6230*	0.5940*
	10	0.1270*	0.5910*	1.0000*	0.5910*	0.5910*	0.4210*	0.5910*	0.5910*	0.5910*	0.5280*	0.5910*
	25	0.0170	0.0480	0.0170	0.0170	0.0170	0.0170	0.1010*	0.0170	1.0000*	0.0480	0.2360*
	50	0.0940*	0.0060	0.0060	0.0060	0.0060	0.0060	0.0940*	0.0060	0.5350*	0.0060	1.0000*
	100	1.0000*	0.4940*	0.5030*	0.5030*	0.5030*	0.4180*	0.7650*	0.5030*	0.6050*	0.9810*	0.9810*
Dec 2011 to Dec 2012 (Low 2)												
4020:4268	5	1.0000*	0.5640*	0.5640*	0.5640*	0.5640*	0.5640*	0.3940*	0.3940*	0.9920*	0.3940*	0.5640*
	10	0.0930*	0.0530*	1.0000*	0.0530*	0.0530*	0.0530*	0.0530*	0.2910*	0.0530*	0.0530*	0.2910*
	25	1.0000*	0.0960*	0.0960*	0.0960*	0.0960*	0.0960*	0.0960*	0.0010	0.3790*	0.0960*	0.0960*
	50	1.0000*	0.1280*	0.3410*	0.3410*	0.3410*	0.3410*	0.0080	0.0000	0.0030	0.0080	0.0050
	100	0.0870*	0.0870*	0.3130*	0.0870*	0.3130*	0.0870*	0.3130*	0.3130*	0.3130*	0.3130*	1.0000*

*Table 4.5: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.*

it performs particularly well during periods of market calm. This confirms the results of Laurent et al. (2012) and those in Chapter 3. In contrast to the entire out-of-sample results of Table 4.2, for the first sub-period of low volatility the VDCC family performs well across the various portfolio sizes. The exception is $N = 5$, where the VDECO family provides the lowest portfolio volatilities in general. CCC provides the lowest standard deviation followed by DCC-ARE and cDCC for the largest portfolios during this period. For the second sub-period of low volatility the VDCC family appears more successful at forecasting the correlation matrix for the largest portfolios, although in the smaller and moderate portfolios the results are mixed with the DEC-TVV and DEC-AVE models performing particularly well. In the large portfolios CCC is again superior under this measure, with the VDCC models giving lower standard deviations relative to the VDECO family. The general success of the VDECO family of models over the entire time period appears to be driven by the subsample of high volatility, as these models perform well across the various portfolio sizes. For the moderate and large portfolios, DEC-TVV and DEC-TVR provide lower standard deviations compared to other methods. As portfolio size increases, the VDCC family broadly performs poorly in the comparison to the VDECO family.

The corresponding MCS results are contained in Table 4.5. All models are included in the MCS for the $N = 5, 10$ and 100 portfolios across all time periods, and follow the trends of Table 4.4. The size of the MCS during the first low volatility sub-period for the 50 asset portfolio is smaller than the other portfolio sizes, with only 5 models included: CCC and the VDCC family, with DCC-AVE excluded. During the second sub-period of low volatility the CCC model is included in the set with a p-value of 1 for $N = 5, 25$ and 50. The sizes of the MCS are in general larger for the second low volatility sub-period. The differences seen between the pre- and post-GFC periods of low volatility again point to a higher level of market volatility overall since the crisis. For the sub-period of high volatility, DECO, DEC-TVV and DEC-TVR are included in the MCS for all portfolios. These results confirm the comparatively good performance of the VDECO family outlined above, suggesting the assumption of equicorrelation is useful during times of market turbulence. This is perhaps due to its tendency to forecast higher peaks and lower troughs than the cDCC-based models.

Overall, in the case of cDCC there is an advantage to conditioning the correlation process on volatility. The volatility dependent models in general lead to better portfolio outcomes compared to the standard cDCC in large portfolios during all time periods. This demonstrates the usefulness of the link between volatility and correlations. However, these differences are not found to be statistically significant. There are differences between the pre-GFC and post-GFC periods of low market volatility, with DCC-AVE having the most success of the VDCC models during and post-GFC. This is in contrast to relatively inferior performance of this method prior to the crisis period. Of the VDECO models, the standard DECO performs well in the large portfolios prior to the GFC but this performance is overtaken by the volatility dependent models (DEC-TVV in particular) both during and following the financial crisis. Interestingly, directly incorporating volatility into the correlations via an additive term appears most effective in the VDCC family. In contrast, for the equicorrelated models a time varying volatility dependence is preferred.

The average value of the constant δ, a measure of the relative economic value of choosing a particular covariance forecasting method over another, is calculated for the various portfolio sizes (see Section 2.6.3 of Chapter 2). The portfolio highlighted here is $N = 50$, contained in Table 4.6. Tables B.7 through B.10 in Appendix B contain the δ values for the remaining four portfolios. If the relative economic value gained from switching from the forecast model in the row heading to that in the column heading is positive, there is an economic advantage in moving from the row model to that in the column. Results reported here assume an expected return of 6% and risk aversion coefficient of $\lambda = 2$.[7] Overall, there appears to be an advantage for switching from a standard cDCC or DECO model to one which conditions on volatility. DEC-TVV performs particularly well in this context against the other VDECO methods across the various portfolio sizes. As N increases DEC-AVE also performs well by this measure. The value of switching from a VDCC method is most pronounced for the $N = 50$ portfolio, with little difference between the various VDECO models. For the largest portfolio, the VDECO family again outperforms the VDCC class although there is an argument of equivalence amongst the equicorrelated models.

[7]No qualitative differences in results were found for expected returns of 8% and 10%, or $\lambda = 5$.

U.S. Equities: Relative economic value, entire period, N = 50

	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	-40.406 0.094	-50.415 0.044	-42.006 0.078	-42.768 0.064	-44.279 0.062	24.192 0.760	29.681 0.782	27.963 0.788	26.310 0.768	25.983 0.776
cDCC		-	-11.027 0.260	-1.657 0.322	-2.498 0.314	-3.985 0.196	62.383 0.908	67.910 0.930	66.152 0.914	64.521 0.906	64.170 0.910
DCC-AVE			-	7.988 0.694	7.196 0.672	5.676 0.658	73.172 0.944	78.760 0.954	76.949 0.952	75.356 0.944	74.969 0.948
DCC-TVV				-	-0.840 0.376	-2.328 0.146	64.003 0.922	69.522 0.940	67.769 0.930	66.137 0.918	65.789 0.924
DCC-ARE					-	-1.528 0.316	64.857 0.924	70.373 0.940	68.623 0.936	66.990 0.928	66.643 0.928
DCC-TVR						-	66.350 0.932	71.873 0.944	70.116 0.940	68.483 0.932	68.136 0.938
DECO							-	5.643 0.796	3.780 0.996	2.196 0.670	1.778 0.940
DEC-AVE								-	-2.014 0.438	-3.579 0.240	-4.024 0.306
DEC-TVV									-	-1.622 0.298	-2.017 0.010
DEC-ARE										-	-0.489 0.474
DEC-TVR											-

Table 4.6: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 50 assets.

Taking the analysis further, the entire sample is split into low and high volatility sub-periods as in the above analysis. These results are included in Tables B.11 through B.25 in Appendix B. During the first low volatility sub-period, CCC and the VDCC family performs well, with the exception of the 10 asset portfolio where VDECO models dominate. This is again true for the second sub-period of relatively low volatility with an advantage in switching to DCC-ARE especially as N increases in size. During the high volatility subsample, VDECO models are generally superior, although this is not the case when $N = 10$ and 25 where DCC-TVR and DCC-AVE provide gains respectively. In general, economic value gains are larger over periods of market turbulence.

4.4 The International Context: European Indices

A portfolio of 14 European indices is used to investigate the usefulness of conditioning the correlation process on volatility in an international context. European countries are chosen to avoid asynchronous trading. All indices are continuously traded over the period 4 June 1996 to 31 December 2014. As in the previous example, log returns are calculated providing a time series of 3919 observations. A list of the countries as well as summary statistics is included in Appendix B. As in the previous example, the VIX is used as the volatility component in the Volatility Dependent models. This is considered to be a reasonable choice,[8] as numerous studies find increasing global integration of equity markets. Berben and Jansen (2005) found correlations between German, UK and U.S. stock markets doubled over the period of 1980 to 2000. Over a period similar to that used here,[9] Sarwar (2014) found a strong negative relationship between the VIX and European stock returns. Additionally, changes in the VIX had significant predictive ability for daily European returns during the recent crisis. Figure 4.6 illustrates the VIX is a reasonable proxy of market volatility, showing the average daily return of the 14 European indices and VIX over the entire period. The correspondence between the two is similar to that seen in Figure 4.1, which showed daily returns of the S&P 500 index and the VIX. Of note here is the last section of observations, representing the addition of 2013 and 2014

[8]Beber, Brandt and Kavajecz (2009) use both the VIX and VSTOXX (a volatility index of the DJ Euro STOXX50 index) as perceived market security risk of the European bond market and find each give similar results.

[9]Sarwar (2014) uses the period 1998 to 2013, defining the ‘crisis period’ as beginning in October 2007.

to the time period. Overall market volatility is relatively low during this time, illustrated both by the VIX and average daily European returns. For the analysis, results are also presented for the period ending 31 December 2012 to enable some comparison between the two empirical applications contained in this chapter.

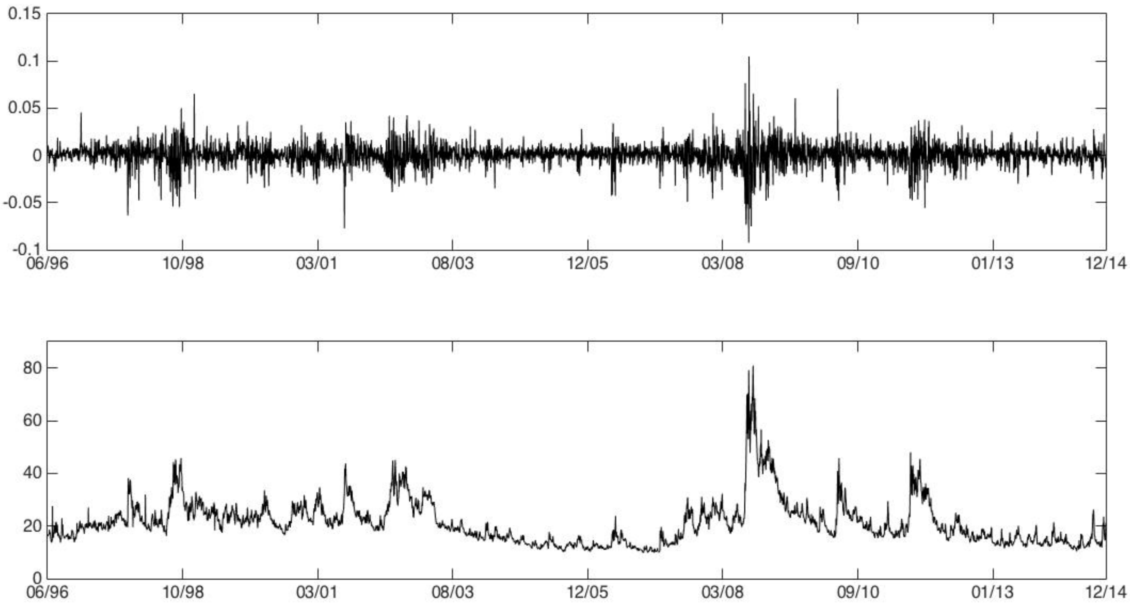

Figure 4.6: Average daily returns, $\bar{r}_t$, of the 14 European indices (top) and level of the VIX (bottom). Period spans 4 June 1996 to 31 December 2014.

4.4.1 Univariate Model Estimation

The volatility process of each country's index is estimated using GJR-GARCH. In the case of the VIX, a two state Markov Switching model is estimated to obtain the expected value of the high volatility regime, $\mathrm{E}_{t-1}(\zeta_{t-1})$. Figure 4.7 shows the VIX along with the filtered probability of being in the high volatility state for the time period considered in this application. Again, the model predicts this 'crisis' regime as having a greater probability more frequently during the GFC.

4.4.2 Full Sample Results

Table 4.7 contains the full sample parameter estimates and log-likelihood values for each of the correlation models, again suppressing the time varying coefficient b_t and logistic function parameters $\theta_{b,0}$ and $\theta_{b,1}$ for the DCC-TVV, DCC-TVR, DEC-TVV and DEC-TVR

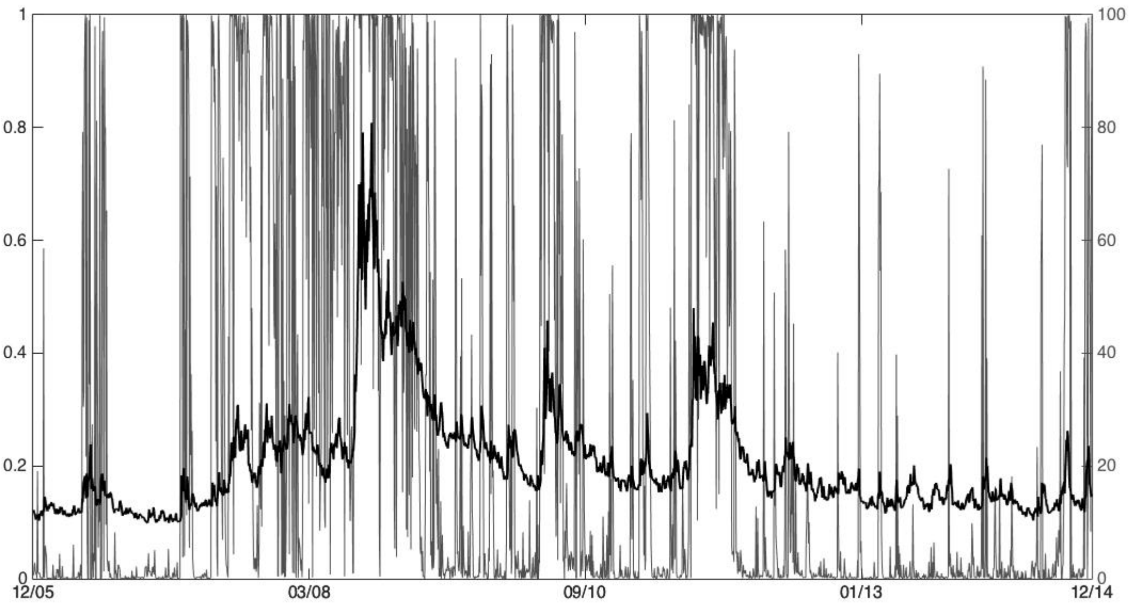

Figure 4.7: Out-of-sample filtered probabilities of high volatility regime of VIX (left axis), as estimated by a two state MS model. VIX over the out-of-sample period (right axis). In sample period is 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.

models. The VDCC and VDECO families provide similar log-likelihood values over the sample. Similar to the U.S. equities example, the addition of the time varying coefficient a_t to the original DECO model seems to result in a decrease in the value of the parameter b and increase in a_t, compared to the constant parameter a. This implies a change in the distribution of past and present information contained in the measure of correlation persistence. Information ranking criteria values, specifically AIC and BIC, are contained in Appendix B (Table B.26) and are consistent with the analysis of the log-likelihoods.

4.4.3 Out of Sample Forecasts

It is useful to compare the DECO and cDCC correlation forecasts to each other, as seen in Figure 4.8. In the cDCC, the pairwise correlations are averaged to give $\bar{\rho}_t$. Figure 4.8 also contains the shared average daily return of the 14 European indices. Both the DECO equicorrelation and cDCC average correlation increase dramatically as variance in the returns increases, falling again during periods of relative calm. The last 500 or so observations exhibit comparatively stable forecasts corresponding to the relatively low volatility

European Indices: Full Sample Results

Model	a	a_0	a_1	b	g	$\theta_{a,0}$	$\theta_{a,1}$	Log-Like
CCC								61067
cDCC	0.0094			0.9807				176568
	(0.0024)			(0.0068)				
DCC-AVE	0.0248			0.8496	−0.0013			176114
	(0.0022)			(0.0194)	(0.0006)			
DCC-TVV		−0.0050	0.0170	0.9946		0.0000	−0.0008	176237
		(0.0013)	(0.0026)	(0.0005)		(0.0030)	(0.0016)	
DCC-ARE	0.0223			0.9638	0.0000			176699
	(0.0166)			(0.0353)	(0.0057)			
DCC-TVR		−0.0046	0.0177	0.9935		0.0000	−0.0022	176690
		(0.0014)	(0.0031)	(0.0008)		(0.0002)	(0.0015)	
DEC	0.0396			0.9425				170050
	(0.0097)			(0.0132)				
DEC-AVE	0.0412			0.9432	0.0007			170116
	(0.0099)			(0.0131)	(0.0005)			
DEC-TVV		0.0615	0.0508	0.8766		0.0005	0.0015	170063
		(0.0299)	(0.1356)	(0.0574)		(0.0445)	(0.1026)	
DEC-ARE	0.0327			0.9510	−0.0089			169970
	(0.0101)			(0.0147)	(0.0019)			
DEC-TVR		0.0615	0.0508	0.8767		0.0005	0.0001	169970
		(0.0570)	(0.0087)	(0.0817)		(0.0035)	(0.0013)	

Table 4.7: Parameter estimates of models for period 4 June 1996 to 31 December 2014 for each correlation model. Robust standard errors in parentheses.

in the returns series during this period. Over the out-of-sample period examined here, the mean level of ρ_t and $\bar{\rho}_t$ appears broadly constant.

The DECO equicorrelation displays higher peaks and lower troughs than the corresponding cDCC measure although both methods rise and fall at the same time. This is unsurprising given the relationship between the two models. Table 4.8 quantifies this result and provides the mean, $\bar{\rho}_t$, and standard deviations for each forecasting method. The $\bar{\rho}_t$ is similar across the VDCC and VDECO models, and the standard deviations are in general lower for the VDCC family than for the VDECO models. The additive regime (ARE) method provides the lowest standard deviation for each group of models respectively.

Dividing the entire out-of-sample period into subsamples based on relative volatility delivers the same trends in results, with the ARE models giving the lowest standard deviations for each model family and the VDCC models providing more stable forecasts regardless of subsample. Also included in Table 4.8 are the same statistics for the period ending 31 December 2012, provided for comparison purposes between the international example presented here and the domestic context in Section 4.3. In general the results are consistent with the entire out-of-sample period, however DECO does give a lower standard deviation than the DCC-TVV and DCC-TVR models in this case.

European Indices: Average correlation forecasts, summary statistics

Entire period	$\bar{\rho}$	s.d.		est. $\bar{\rho}$	s.d.
DECO	0.5401	0.0799	cDCC	0.5341	0.0648
DEC-AVE	0.5429	0.0920	DCC-AVE	0.5450	0.0648
DEC-ARE	0.5193	0.0647	DCC-ARE	0.5239	0.0592
DEC-TVV	0.5322	0.0929	DCC-TVV	0.5369	0.0664
DEC-TVR	0.5320	0.0925	DCC-TVR	0.5339	0.0649
Sub-periods	$\bar{\rho}$	s.d.		est. $\bar{\rho}$	s.d.
Dec 2005 to Jul 2007 - Low 1 (2001:2359)					
DECO	0.4850	0.0719	cDCC	0.4767	0.0540
DEC-AVE	0.4433	0.0732	DCC-AVE	0.4916	0.0574
DEC-ARE	0.4695	0.0503	DCC-ARE	0.4719	0.0505
DEC-TVV	0.4715	0.0855	DCC-TVV	0.4915	0.0733
DEC-TVR	0.4707	0.0837	DCC-TVR	0.4777	0.0578
Jul 2007 to Dec 2011 - High (2360:3265)					
DECO	0.5869	0.0725	cDCC	0.5735	0.0558
DEC-AVE	0.6015	0.0771	DCC-AVE	0.5840	0.0547
DEC-ARE	0.5544	0.0622	DCC-ARE	0.5569	0.0536
DEC-TVV	0.5793	0.0912	DCC-TVV	0.5737	0.0564
DEC-TVR	0.5791	0.0909	DCC-TVR	0.5726	0.0554
Dec 2011 to Dec 2014 - Low 2 (3266:3917)					
DECO	0.5055	0.0521	cDCC	0.5110	0.0437
DEC-AVE	0.5164	0.0523	DCC-AVE	0.5201	0.0454
DEC-ARE	0.4979	0.0447	DCC-ARE	0.5066	0.0402
DEC-TVV	0.5002	0.0608	DCC-TVV	0.5109	0.0438
DEC-TVR	0.5002	0.0608	DCC-TVR	0.5111	0.0442
End 2012 (2001:3495)					
DECO	0.5549	0.0829	cDCC	0.5459	0.0679
DEC-AVE	0.5537	0.0997	DCC-AVE	0.5571	0.0667
DEC-ARE	0.5302	0.0677	DCC-ARE	0.5335	0.0627
DEC-TVV	0.5448	0.0979	DCC-TVV	0.5495	0.0979
DEC-TVR	0.5445	0.0975	DCC-TVR	0.5456	0.0975

Table 4.8: Out-of-sample mean, $\bar{\rho}$, and standard deviation for each equicorrelation model and average correlations of cDCC models for portfolio of 14 European indices. In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.

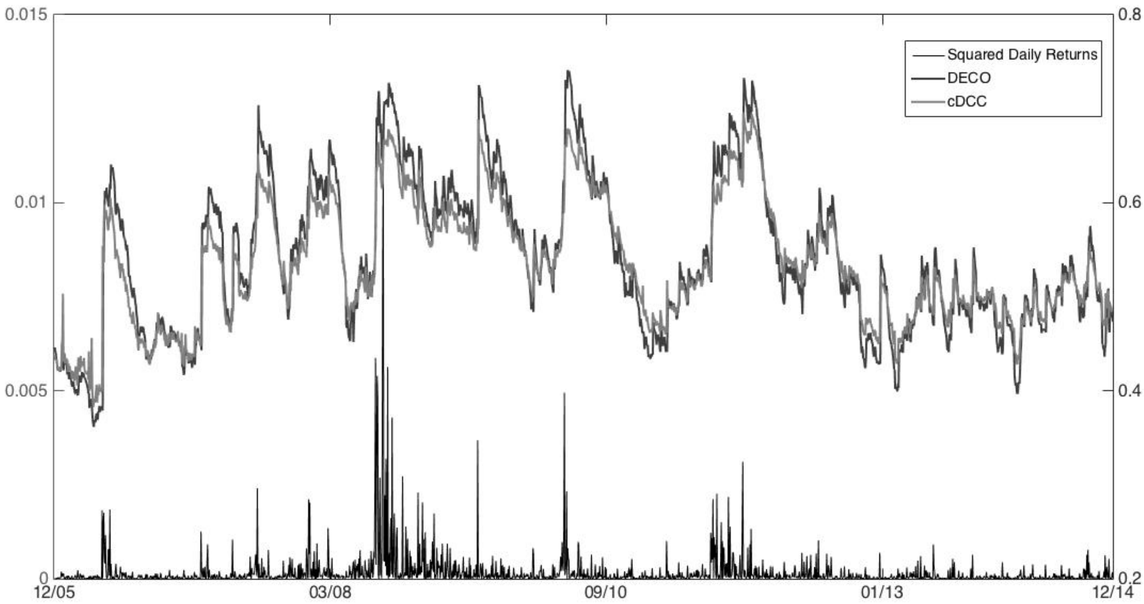

Figure 4.8: One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for cDCC and one-step-ahead equicorrelation, ρ_t, of DECO (left axis). Squared average daily return of 14 European indices for out-of-sample period (right axis). In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.

To visualise the similarities and differences between the various equicorrelation forecasts Figure 4.9 shows the 1917 one-step-ahead forecasts of equicorrelation, ρ_t, for each of the VDECO models. The DEC-TVV method demonstrates the highest variation in equicorrelations across the sample, with higher peaks and lower troughs than the other methods. The equicorrelation forecasts are relatively different to each other for the first 500 or so forecasts, becoming more similar across the rest of the out-of-sample period.

Turning attention to the VDCC family of models, Figure 4.10 shows the out-of-sample average correlation forecasts for each of the VDCC methods, finding patterns similar to the VDECO forecasts. The DCC-TVV seems to be more variable than the other models although not to the same extent as in the DEC-TVV case.

Table 4.9 provides the annualised volatilities of the GMV portfolios generated using each method and corresponding MCS results for the entire out-of-sample period, the subsamples based on relative high and low volatility and the period ending 2012. By this measure the VDECO family performs poorly in comparison to the VDCC and this result is in contrast to those provided in Section 4.3 in the context of U.S. equities. It appears the assumption of equicorrelation is not useful for portfolio allocation purposes

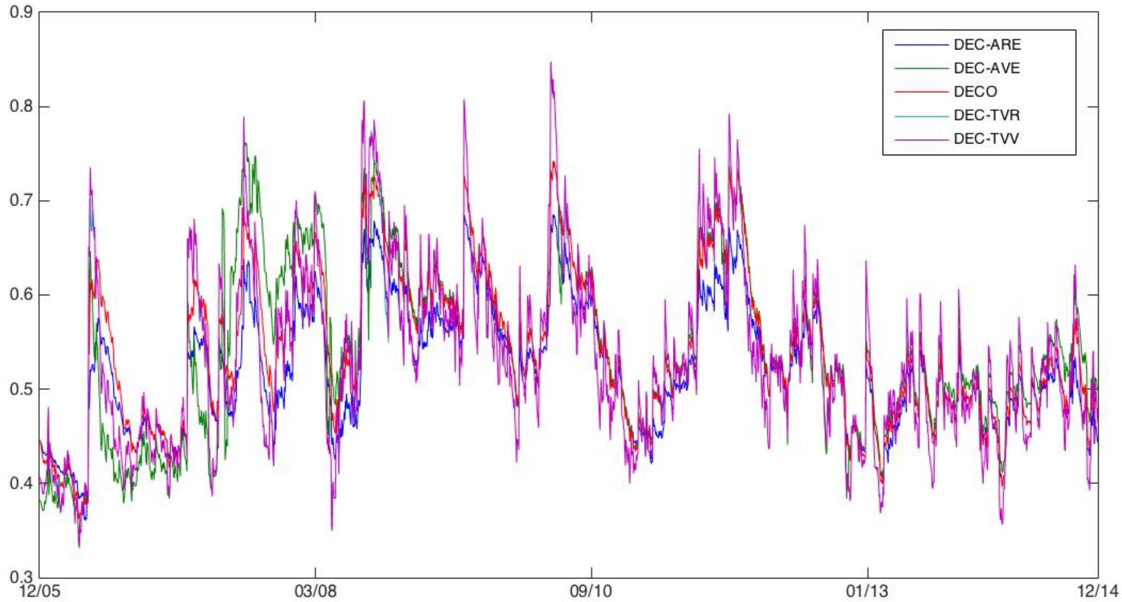

Figure 4.9: One-step-ahead forecasts of equicorrelation, ρ_t, for the DECO-based models. In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.

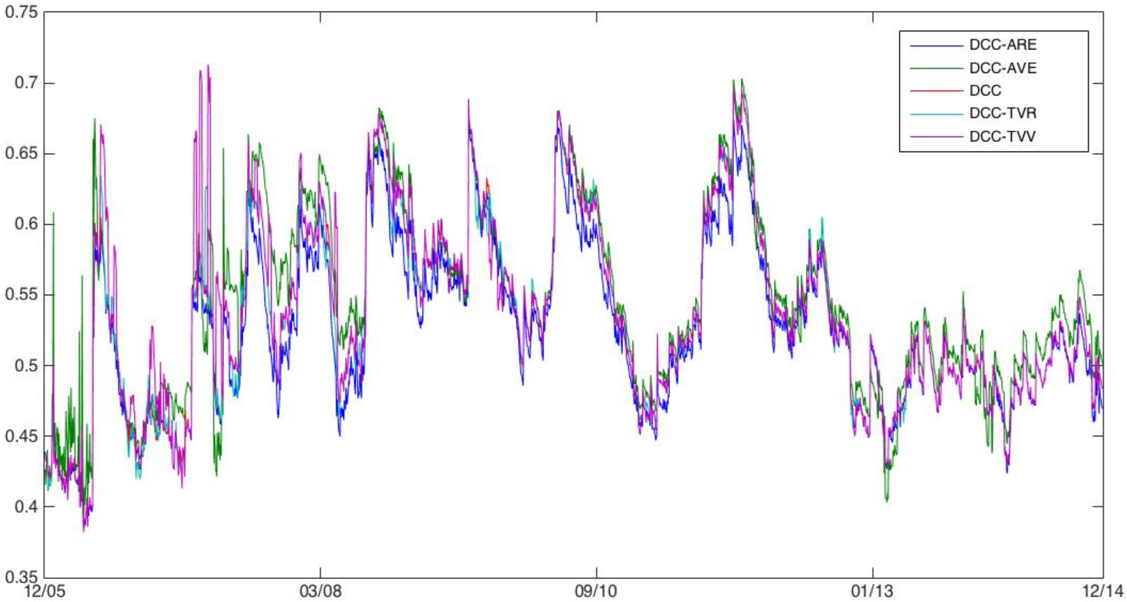

Figure 4.10: One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC-based models. In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.

European Indices: Forecasting results

	CCC	cDCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
Entire sample											
% Vol.	16.1405	13.2750	13.3184	13.2554	13.3090	13.8197	17.6391	17.6696	17.6335	17.5979	17.6315
MCS	0.0120	0.4590*	0.4590*	1.0000*	0.4590*	0.4590*	0.0000	0.0000	0.0000	0.0000	0.0000
Dec 2005 to Jul 2007 - Low 1 (2001:2359)											
% Vol.	10.8699	9.4479	9.1778	9.1741	9.4965	13.0616	12.1421	11.9204	11.9856	12.1421	11.9956
MCS	0.0050	0.0140	0.9100*	1.0000*	0.0140	0.0050	0.0000	0.0000	0.0000	0.0000	0.0000
Jul 2007 to Dec 2011 - High (2360:3265)											
% Vol.	21.0658	17.1610	17.2827	17.1936	17.2149	17.1552	22.6030	22.6851	22.6255	22.5600	22.6201
MCS	0.0150	0.4790*	0.0950*	0.4790*	0.4790*	1.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
Dec 2011 to Dec 2014 - Low 2 (3266:3917)											
% Vol.	9.0524	7.6075	7.6247	7.6047	7.5804	7.5898	10.9085	10.9452	10.9087	10.8475	10.9088
MCS	0.0000	0.0010	0.0010	0.0050	1.0000*	0.4570*	0.0000	0.0000	0.0000	0.0000	0.0000
End 2012 (2001:3495)											
% Vol.	17.7087	14.5244	14.5749	14.5011	14.5665	15.1605	19.2054	19.2295	19.1882	19.1666	19.1859
MCS	0.0130	0.4470*	0.4280*	1.0000*	0.4280*	0.4280*	0.0010	0.0010	0.0010	0.0010	0.0010

*Table 4.9: Annualised percentage volatility of out-of-sample global minimum variance (GMV) portfolio returns for each equicorrelation measure for portfolio of 14 European indices. Empirical MCS of the out-of-sample GMV portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence. In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.*

when considering an international portfolio of assets. This finding is explored further in Section 4.5.

Of the VDECO methods, DEC-ARE provides the lowest standard deviation in all periods except the first low volatility subsample, where the DEC-AVE method generates the lowest portfolio standard deviation. The CCC method provides lower volatilities than the VDECO family in all cases although it is not included in the MCS for any of the time periods considered. In the case of the VDCC models, all VDCC models are contained in the MCS for the entire out-of-sample period with DCC-TVV included in the set with a p-value of 1. This is also the case for the period ending 2012. All VDCC methods are contained in the MCS for the high volatility subsample. For the periods of relatively low volatility, only two models are contained in each MCS and these are different between the two sub-periods. For the first low volatility subsample, the included methods are DCC-TVV and DCC-AVE. Recall these are the models using the level of the VIX as the volatility component. In the second low volatility subsample, it is the models using the regime of volatility, DCC-ARE and DCC-TVR that are contained in the MCS. It is conjectured that this difference between periods of market calm implies that the post-GFC world is quite different to that seen pre-crisis.

Lastly, the economic value of switching from one volatility timing approach to another is investigated. Tables B.27 through B.31 (in Appendix B) report the average value of the constant δ, a measure of the relative economic value of choosing a particular covariance forecasting method over another. If the relative economic value gained by switching from the forecast model in the row heading to that in the column heading is positive, there is an economic advantage in moving from the row model to that in the column. As was the case previously, results reported here assume an expected return of 6% and risk aversion coefficient of $\lambda = 2$. No qualitative differences in results were found for expected returns of 8% and 10%, or $\lambda = 5$. With the exception of the high volatility sub-period, the DCC-AVE model is preferred using this measure and provides an economic gain if switched to from a competing specification. During the high volatility sub-period the DCC-TVV model provides an advantage. Across the various time periods, the VDECO family performs poorly in this context.

4.5 Domestic vs. International Data and Equicorrelation

The consistently strong performance of equicorrelated models in the domestic application of U.S. equities and the relatively poor performance for the portfolio of European indices warrants further discussion. A starting point is further analysis of the VDECO class of model in the context of the European portfolio. Table 4.10 contains the MCS results of the equicorrelated models only, illustrating differences across the varying subsamples previously hidden by the superior performance of the VDCC models. The standard DECO model is excluded from the MCS during both sub-periods of low volatility, providing support for conditioning the equicorrelation structure on volatility. The preferred specification is not clear, as the DEC-ARE model is included in the MCS for all periods except the first sub-period of low volatility. The exclusion of DEC-ARE for this low volatility sub-period is in contrast to its relative success over the remainder of the sample. However, this analysis does not adequately explain the comparatively poor performance of the equicorrelated models in the international context, relative to that seen in a single national market. It is worthwhile noting that N is comparatively smaller for the European dataset than in the U.S. equities example, where DECO performed relatively well. Portfolio size may certainly be a factor in these results, however it is unlikely to drive the entire discrepancy between cDCC and DECO in the international setting by itself. It is therefore conjectured in this chapter the answer concerns the use of historical information in the cDCC and DECO frameworks.

Engle and Kelly (2012, p. 213) explain DECO's advantage over DCC, stating

> *"To the extent that true correlations are affected by realisations of all assets, the failure of DCC to capture the information pooling aspect of DECO can disadvantage DCC as a descriptor of the data-generating process."*

Perhaps the opposite is true in the case of market indices, as used in the international example above. The information pooling advantage DECO has over cDCC in the context of equity returns, such as those in the domestic U.S. example, do not exist in the case of an index. By construction, the information of individual constituents of the index has already been pooled. This erosion of DECO's advantage leads to the conclusion that the ability of cDCC to track the dynamics of the correlation process is more useful in this setting. This has certainly been demonstrated in the forecasting applications undertaken in this study and provides guidance to practitioners regarding model selection.

European Indices: MCS, VDECO only

	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
Entire sample					
MCS	0.4130*	0.4100*	0.8630*	1.0000*	0.8630*
Dec 2005 to Jul 2007 - Low 1 (2001:2359)					
MCS	0.0140	1.0000*	0.2730*	0.0210	0.2620*
Jul 2007 to Dec 2011 - High (2360:3265)					
MCS	0.7400*	0.2190*	0.7400*	1.0000*	0.7400*
Dec 2011 to Dec 2014 - Low 2 (3266:3917)					
MCS	0.0360	0.0360	0.1130*	1.0000*	0.0360
End 2012 (2001:3495)					
MCS	0.7040*	0.7040*	0.8940*	1.0000*	0.8940*

*Table 4.10: Empirical MCS of out-of-sample global minimum variance portfolio for portfolio of 14 European indices, VDECO models only. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence. In-sample period of 2000 observations (Jun 1996 to Dec 2005), entire period spans 4 June 1996 to 31 December 2014.*

4.6 Conclusion

This chapter provides the equicorrelation equivalent of the Volatility Dependent Dynamic Conditional Correlation (VDCC) models of Bauwens and Otranto (2013), who show that conditioning correlations on volatility is worthwhile. The DECO model is extended based on their framework. Out-of-sample forecasting performance of the Volatility Dependent Dynamic Equicorrelation (VDECO) models is compared to the VDCC through forming global minimum variance (GMV) portfolios, Model Confidence Sets where the loss function is the squared GMV portfolio returns, and the explicit economic value of switching from one method to another. The out-of-sample period is also broken into subsamples of high and low relative market volatility to further the evaluation of forecasts. This methodology is applied to two datasets, the first comprised of U.S. equities and the second European indices.

For the domestic application of U.S. equities, the VDECO models generally provide lower variances than the VDCC family of models as portfolio size increases. Directly conditioning correlations on volatility via an additive term appears most effective in the VDCC family, in contrast to the equicorrelated models where a time varying volatility dependence is preferred. Differences in forecasting ability over periods of low and high volatility points to a higher level of market volatility since the global financial crisis (GFC). VDECO performs well over this period of market turbulence. For the equicorrelated models, the standard DECO performs well for the large portfolios prior to the GFC but this performance is overtaken by the volatility dependent models both during and following the financial crisis. This result is worth emphasising, as correlation forecasting is of particular interest during times of market turbulence and certainly VDECO appears to be worthwhile in this context. Furthermore, evidence regarding the usefulness of a simplistic model such as CCC is found, specifically for large portfolios and periods of market calm, although this result is not statistically significant.

The second empirical application is a set of 14 European market indices. For this international example, the equicorrelated models perform poorly against the cDCC-based methods, across all metrics used in this chapter. Regarding the VDCC models, there is a definite advantage in extending the standard cDCC framework to condition on volatility although which is the best specification to use varies over the sample. This again points to a different post-GFC world than that seen pre-GFC. The contrasting results given by the domestic and international datasets provides insight into what drives the success of an equicorrelation model over the cDCC. Reasoning presented here concludes the benefit of information pooling allows the DECO framework to enjoy an advantage over the cDCC model for a portfolio of equities, however this advantage is eroded in the case of market indices. By construction, a market index has pooled the information of individual constituents and thus allows the cDCC to more accurately model the correlation dynamics of such a portfolio. This provides those seeking to model volatility and correlations with further information regarding model selection when forming forecasts.

Overall, conditioning correlations on volatility for the VDCC-based models is helpful. The DECO framework appears to benefit comparatively less from a volatility dependent structure. The differences in what structure the volatility dependence should take across various sub-periods provides scope for future work. In order to best exploit volatility as a determinant of correlations, further research into the nature of the linkage between volatility and correlations is needed to consistently form superior correlation forecasts.

The next chapter seeks to further investigate and model the correlation dynamics of equities. The complexities of modelling the intraday correlations of high frequency returns data are investigated and an MGARCH approach to capture these effects is presented. Insights into the dynamics of the intraday correlations process for a portfolio of equities are provided and ideas for further work in this relatively new area of financial econometrics are suggested.

Chapter 5

Modelling Intraday Correlations using Multivariate GARCH

5.1 Introduction and Motivation

The aim of this chapter is to develop a modelling framework for the intraday correlation matrix, examining the correlation dynamics of a portfolio of equities at a high frequency.[1] The study of high frequency correlations is motivated by a large number of practical financial applications, with the requirement of institutions such as banks and hedge funds to have up-to-date risk profiles for their portfolios. Uses for intraday correlation forecasts include hedging, the scheduling of trades and setting of limit orders.

In contrast to the volatility process of an individual asset discussed in Section 2.4.5, pairwise correlations of a portfolio of assets appear to display an inverted U-shaped pattern over the trading day. Patterns in intraday correlations have been noted in the literature, see Section 2.5.5. The approach detailed in this chapter is quite different to previous studies in this area, examining the correlation dynamics over the trading day with the specific aim of modelling these processes. The models presented in this chapter are based on the consistent DCC (cDCC) model of Aielli (2013) and the DECO model of Engle and Kelly (2012), adapted to capture both the daily persistence and the intraday inverted U-shape pattern seen in the correlations between assets over the trading day.

[1] The idea of intraday correlations is distinct to the 'realized covariance' or *RCOV* literature, that is using intraday data sampled at high frequencies for the purposes of generating daily covariance or correlation matrices. The focus here is modelling intraday correlations using intraday data.

Estimation results indicate modelling the diurnal pattern in correlations over the trading day is potentially useful, in a similar way to the importance of accounting for diurnal patterns seen in volatilities. The analysis also highlights the relevance of daily persistence in correlations, with the models allowing for both the intraday pattern in correlations and daily level persistence in correlations providing promising results in terms of fit over the sample. A further examination of sub-portfolios based on industry reveals the intraday pattern in the correlations is most evident between stocks that have a lower level of unconditional correlation, such as those from different industries. Stocks that are highly correlated display the pattern, however it is not as pronounced.

5.2 Methodology

Throughout the empirical work contained in Chapters 3 and 4, the focus has been forecasting the conditional correlation matrix at a daily frequency, with a portfolio allocation exercise as part of the evaluation framework. The various estimators for the daily conditional correlation matrix, $\mathbf{R}_t$, are used as an input into the decomposition of the conditional covariance matrix, $\mathbf{H}_t$, in equation 3.1. This decomposition is extended to the intraday context here as

$$\mathbf{H}_{t,i} = \mathbf{D}_{t,i}\mathbf{R}_{t,i}\mathbf{D}_{t,i} \ , \tag{5.1}$$

where $\mathbf{R}_{t,i}$ is the intraday conditional correlation matrix and $\mathbf{D}_{t,i}$ is the diagonal matrix of intraday conditional standard deviations of the returns on day t for intraday interval i. As is the case at the daily frequency, $\mathbf{H}_{t,i}$ is estimated in two stages: firstly, the univariate standard deviations of $\mathbf{D}_{t,i}$ and, secondly the correlations between assets contained in $\mathbf{R}_{t,i}$. This section details the model used to estimate the univariate intraday volatility process of each asset in the portfolio, before describing the framework used to model the intraday correlations.

5.2.1 Intraday Univariate Volatility

The univariate framework used to estimate the individual volatility process of each stock is based on the multiplicative component GARCH model of Engle and Sokalska (2012). Recall from Section 2.4.5 of Chapter 2 that this approach decomposes the volatility of high frequency returns into daily, diurnal and intraday variances (see equation 2.35). The estimation procedure involves modelling the daily variance, h_t, then standardising the intraday returns in order to estimate the diurnal pattern, s_i. The returns are then conditioned by the diurnal component with an univariate GJR–GARCH model to capture the remaining intraday persistence.

Engle and Sokalska (2012) used commercially available volatility forecasts for h_t based on a risk factor model, however in this chapter the daily variance is linked to the lagged volatility of the previous day. This approach allows for the use of the realized volatility, $RV_t = \sum_{i=0}^{I} r_{t,i}^2$, and does not require selection of any common risk factors (as in Engle and Sokalska, 2012). The AR(1) used here is

$$h_t = \mu + \varphi RV_{t-1} \ , \tag{5.2}$$

where RV_{t-1} is the realized volatility on day $t-1$, μ the unconditional volatility and φ a scaling parameter.

The intraday returns are scaled by the daily variances, allowing for the intraday diurnal pattern in the returns, s_i, to be modelled using equation 2.36. Equation 2.37 specifies the returns, $z_{t,i}$, conditioned by both the daily variance and diurnal components. The residual intraday variance is then modelled using a GJR–GARCH(1,ϕ,1) specification

$$q_{t,i} = \omega + \alpha z_{t,i-1}^2 + \phi z_{t,i-1}^2 I[z_{t,i-1} < 0] + \beta q_{t,i-1} \ . \tag{5.3}$$

Here, $\omega = (1 - \alpha - \beta - \phi/2)$ and $I[z_{t,i-1} < 0]$ is a dummy indicator variable that takes the value 1 if $z_{t,i-1}$ is negative and 0 otherwise. The usual constraints apply, that is $\omega > 0$, $\alpha + (\phi/2) \geq 0$, $\beta \geq 0$ and $\alpha + (\phi/2) + \beta < 1$. To summarise, the parameters estimated for the multiplicative component GARCH are $[\mu, \varphi, \alpha, \beta, \phi]$.

5.2.2 Intraday Dynamic Conditional Correlation

Recall from earlier chapters the cDCC conditional correlation matrix, $\mathbf{R}_t$ in equation 2.46, is a scaled version of the pseudo-correlation matrix, $\mathbf{Q}_t$. The time series, $t = 1, \ \ldots \ , T$, is sampled at the daily frequency. In this chapter, the concern is correlation dynamics over the trading day. Time is now denoted as day t and intraday interval i.

For the purposes of modelling intraday conditional correlations the cDCC specification is now defined as

$$\mathbf{R}_{t,i} = \mathrm{diag}(\mathbf{Q}_{t,i})^{-1/2}\ \mathbf{Q}_{t,i}\ \mathrm{diag}(\mathbf{Q}_{t,i})^{-1/2}\ . \tag{5.4}$$

Several new specifications for the pseudo-correlation matrix, $\mathbf{Q}_{t,i}$, are provided for modelling pairwise intraday correlations. The first is simply the original cDCC model given in equation 3.2, applied at an intraday frequency rather than daily

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1-a-b) + a\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}'_{t,i-1}\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b\ \mathbf{Q}_{t,i-1}\ , \tag{5.5}$$

where $\bar{\mathbf{Q}}$ is the unconditional sample correlation of volatility standardised returns, a and b are parameters subject to the positivity constraints $a > 0$, $b > 0$ and $a + b < 1$, and $\hat{\boldsymbol{\epsilon}}_{t,i-1}$ the vector of volatility standardised returns for day t, interval $i-1$. As the parameters here are scalar values, the correlation dynamics are the same for all assets. For the purposes of this chapter this model is referred to as cDCC and will represent a benchmark to which the following extensions are compared.

Allez and Bouchaud (2011, p. 11) find *"... average correlation between stocks increases throughout the day ... "* and this is later confirmed by Tilak et al. (2013). Here, the suggestion of a diurnal pattern in correlations over the trading day provides further confirmation. Certainly, a model accounting for any intraday pattern in the pairwise conditional correlation processes is desirable. In equation 5.5, the pseudo-correlation is mean reverting to the unconditional correlation, $\bar{\mathbf{Q}}$. The approach taken here is in the spirit of how the intraday diurnal pattern is captured in the univariate case for the Engle and Sokalska (2012) method, described above. In the following DCC-Intraday model, the intention is to allow the intraday correlation to revert to the diurnal pattern seen in the pairwise correlations over the trading day, shown as

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}_i^{DI}(1-a-b) + a\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}'_{t,i-1}\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b\ \mathbf{Q}_{t,i-1}\ . \tag{5.6}$$

The parameters a and b are subject to the same constraints as in equation 5.5. The matrix $\bar{\mathbf{Q}}_i^{DI}$ is the outer product of standardised returns for each 5-minute interval i of the trading day, averaged over the T days and scaled to give a $N \times N$ correlation matrix for each of the I intervals,

$$\bar{\mathbf{Q}}_i^{DI} = \bar{\mathbf{Q}}_i^* \left(\frac{1}{T} \sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_{t,i} \hat{\boldsymbol{\epsilon}}'_{t,i} \right) \bar{\mathbf{Q}}_i^* \, . \tag{5.7}$$

Here, $\bar{\mathbf{Q}}_i^* = \text{diag} \left(\frac{1}{T} \sum_{t=1}^{T} \hat{\boldsymbol{\epsilon}}_{t,i} \hat{\boldsymbol{\epsilon}}'_{t,i} \right)^{-1/2}$.

A similar technique can be used to account for correlation persistence at the daily level and the intent is to revert to a daily correlation as in

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}_{t-1}^{DY}(1-a-b) + a \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t,i-1} \hat{\boldsymbol{\epsilon}}'_{t,i-1} \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b \, \mathbf{Q}_{t,i-1} \, . \tag{5.8}$$

Here $\bar{\mathbf{Q}}_t^{DY}$ is the outer product of standardised returns, averaged over the I intervals of the trading day t and scaled to give a $N \times N$ correlation matrix for each of the T days,

$$\bar{\mathbf{Q}}_t^{DY} = \bar{\mathbf{Q}}_t^* \left(\frac{1}{I} \sum_{i=1}^{I} \hat{\boldsymbol{\epsilon}}_{t,i} \hat{\boldsymbol{\epsilon}}'_{t,i} \right) \bar{\mathbf{Q}}_t^* \, . \tag{5.9}$$

Here, $\bar{\mathbf{Q}}_t^* = \text{diag} \left(\frac{1}{I} \sum_{i=1}^{I} \hat{\boldsymbol{\epsilon}}_{t,i} \hat{\boldsymbol{\epsilon}}'_{t,i} \right)^{-1/2}$. The parameters a and b are subject to the same constraints as in equation 5.5. Referred to as DCC-Daily I, this is the first of three specifications incorporating persistence in the correlation dynamics at the daily level.

The second model, or DCC-Daily II, accounting for daily persistence when modelling intraday correlations is

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1-a-c) + a \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t,i-1} \hat{\boldsymbol{\epsilon}}'_{t,i-1} \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + c \, \bar{\mathbf{Q}}_{t-1}^{DY} \, . \tag{5.10}$$

The correlation is mean reverting in the sense of the original cDCC, that is reverting to the unconditional $\bar{\mathbf{Q}}$. The previous day's daily level correlation, $\bar{\mathbf{Q}}_{t-1}^{DY}$, enters the model. The scaling parameter c is constrained to be positive, $c > 0$, to ensure positive definiteness, and $a + c < 1$.

The third model, or DCC-Daily III, is an unrestricted version of equation 5.10. Here, both the previous interval's pseudo-correlation, $\mathbf{Q}_{t,i-1}$, as well as the additive term for the persistence in the daily correlations, $\bar{\mathbf{Q}}_{t-1}^{DY}$, are included

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}(1-a-b-c) + a \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} \hat{\boldsymbol{\epsilon}}_{t,i-1} \hat{\boldsymbol{\epsilon}}'_{t,i-1} \, \text{diag}(\mathbf{Q}_{t,i-1})^{1/2} + b \, \mathbf{Q}_{t,i-1} + c \, \bar{\mathbf{Q}}_{t-1}^{DY} \, . \tag{5.11}$$

Here, $a > 0$, $b > 0$, $c > 0$ and $a + b + c < 1$.

The final model is designed to account for both persistence in the daily correlations and the diurnal pattern evident over the trading day, in the spirit of the full univariate model of Engle and Sokalska (2012). Given the importance of capturing both the intraday diurnal pattern and daily-level variance in the univariate case, it is reasonable to expect the two effects will be important in the correlation context. DCC-Both includes the intraday correlation $\bar{\mathbf{Q}}_i^{DI}$ as the intercept, accounting for the daily level persistence additively with the term $c\,\bar{\mathbf{Q}}_{t-1}^{DY}$,

$$\mathbf{Q}_{t,i} = \bar{\mathbf{Q}}_i^{DI}(1 - a - c) + a\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2}\hat{\boldsymbol{\epsilon}}_{t,i-1}\hat{\boldsymbol{\epsilon}}'_{t,i-1}\ \mathrm{diag}(\mathbf{Q}_{t,i-1})^{1/2} + c\,\bar{\mathbf{Q}}_{t-1}^{DY}\ . \tag{5.12}$$

The parameters are constrained to be positive, $a > 0$ and $c > 0$, and $a + c < 1$. This allows the conditional correlations to revert to the intraday pattern, whilst capturing the daily level persistence of the correlations. The specification omits the $\mathbf{Q}_{t,i-1}$ term, representing the relationship of the previous interval's correlation to the current correlation. Preliminary experiments found that the addition of both intraday and daily level correlation terms rendered this variable redundant.

5.2.3 Intraday Dynamic Equicorrelation

All the cDCC-based models above readily extend to the equicorrelation context. The assumption of equicorrelation has been found to be useful in the context of modelling correlations at the daily frequency, as outlined in previous chapters. It is reasonable to conjecture that similar benefits may exist at the intraday frequency and subsequently the equicorrelated models are included in the analysis.

The DECO framework using intraday data is shown as

$$\rho_{t,i} = \frac{1}{N(N-1)}\left(\mathbf{1}'\mathbf{R}_{t,i}^{DCC}\mathbf{1} - N\right) = \frac{2}{N(N-1)}\sum_{n>m}\frac{q_{n,m,t,i}}{\sqrt{q_{n,n,t,i}q_{m,m,t,i}}} \tag{5.13}$$

where $q_{n,m,t,i}$ is the n, mth element of the pseudo-correlation matrix $\mathbf{Q}_{t,i}$ using equation 5.5. Similarly, the intraday diurnal pattern in the correlations as well as a daily persistence variable can be included in the conditional pseudo-correlations as described above. Subsequently the equicorrelations are formed using equation 5.13, with the relevant specification of $\mathbf{Q}_{t,i}$. In keeping with the terminology used previously these models are referred to as DECO, DECO-Intraday, DECO-Daily I, DECO-Daily II, DECO-Daily III, DECO-Both.

5.3 Data

The dataset contains 5-minute returns of five stocks traded on the Australian Stock Exchange (ASX)[2] over the period 4 January 2011 to 29 December 2012. The companies are ANZ, BHP, NAB, RIO and WOW representing two banks, two resource companies and one retailer. There are 34,720 5-minute observations over 496 trading days, with 70 5-minute intervals per trading day. Trading begins at 10:10 AM and finishes at 4:00 PM, Monday to Friday. The market technically opens at 10:00 AM, however common practice is to discard the first 10 minutes of the trading day when using ASX data. This avoids the opening auction period of the ASX used by the exchange to determine opening prices, see Hall and Hautsch (2006), among many others.

Intraday returns are generated as $r_{t,i} = \log(C_{t,i}/C_{t,i-1})$ where $C_{t,i-1}$ and $C_{t,i}$ are the closing prices of interval $i-1$ and i on day t. The exception is the first period of the day, when the price at the opening of the 10:10 AM - 10:15 AM interval is used to generate the return $r_{t,1}$, that is $C_{t,i-1} = O_{t,1}$. Figure 5.1 shows the intraday returns for each of the five stocks over the sample period. A period of high volatility common to all stocks is observed from June 2011 to September 2011 and corresponds to the downgrading of US credit in response to the European debt crisis.

A common feature in all measures of intraday trading is a diurnal pattern in the volatility process. This U-shape is documented by many researchers, see Andersen and Bollerslev (1997) and Engle and Sokalska (2012) among others. It is easily seen in the average squared intraday returns for each stock, $\bar{r}_i^2$, as in Figure 5.2. The squared returns series $r_{t,i}^2$ has been averaged across the t days for each i to generate $\bar{r}_i^2$. Evidence of this pattern in the volatility process of equity returns sampled at a high intraday frequency has complicated modelling of these processes.

Prior to any formal analysis of intraday correlations, it is useful to examine simple unconditional correlations (Table 5.1), using the raw returns $r_{t,i}$. ANZ and NAB are both

[2] ASX data, as opposed to the U.S. or European, is used in this chapter as it was the most reliable source of high frequency data available.

Intraday Returns

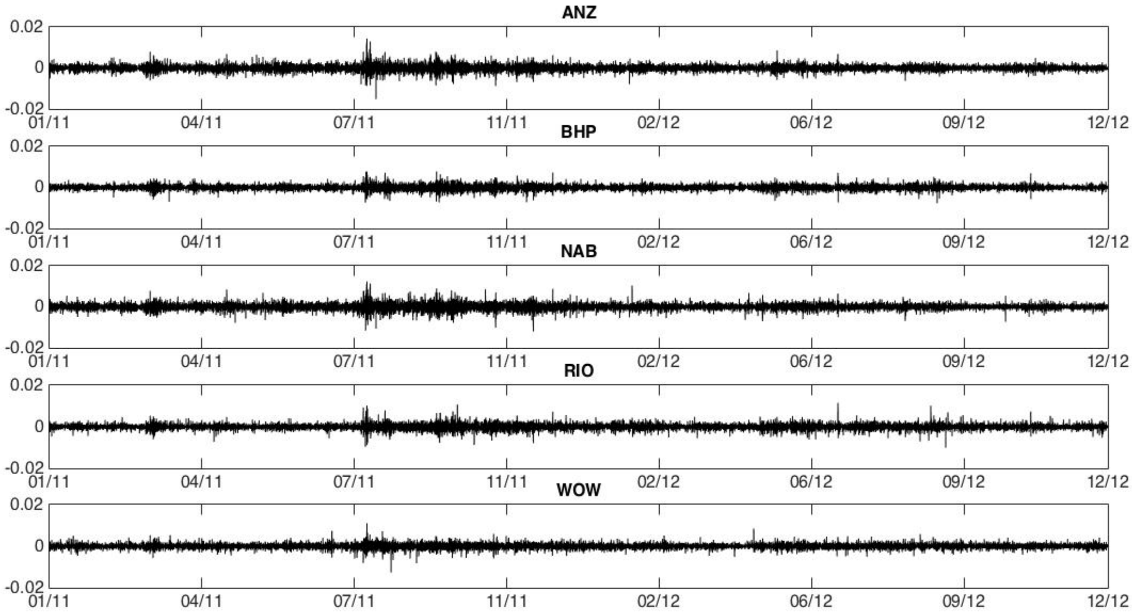

Figure 5.1: 5-minute intraday returns of each of the 5 Australian equities, entire period spans 4 January 2011 to 29 December 2012.

Average Squared Returns

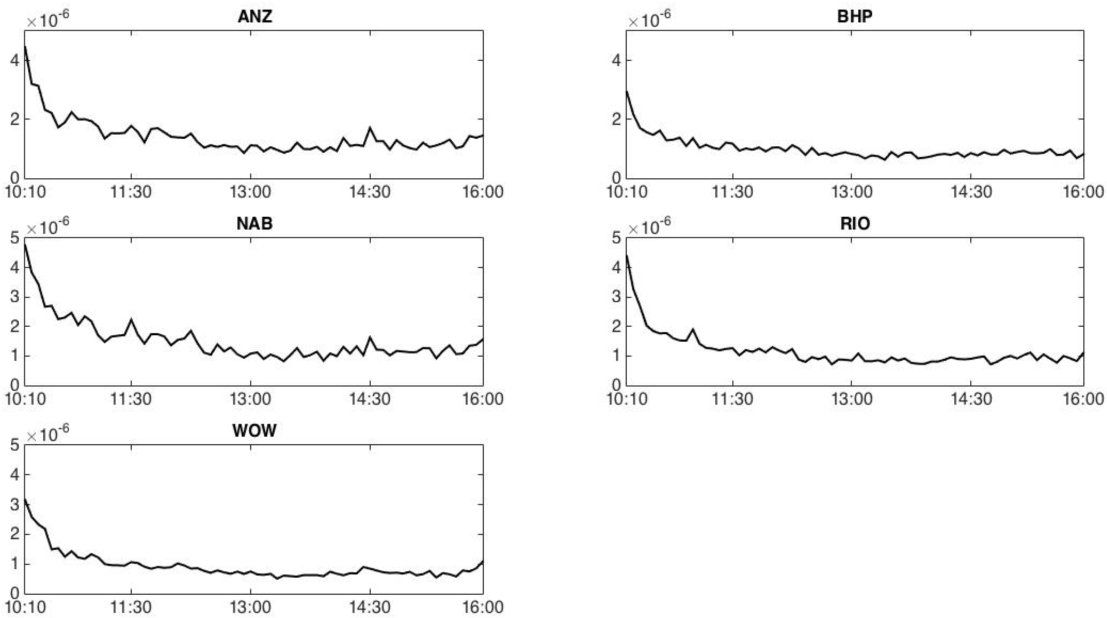

Figure 5.2: Average squared 5-minute intraday returns of each stock, $\bar{r}_i^2$, entire period spans 4 January 2011 to 29 December 2012.

banking stocks; BHP and RIO belong to the resource sector; and, WOW is the lone retailer in the dataset. As would be expected, the correlations are higher for those stocks from the same industry. The pair of resource companies are more highly correlated with the banking pair than they are with WOW. In the analysis contained in the following section, these between- and within-industry differences are explored in terms of the effect (if any) on the behaviour of the correlation dynamics of the portfolio.

Unconditional Correlations

Industry	Banking		Resources		Retail
ρ	ANZ	NAB	BHP	RIO	WOW
ANZ	–	0.6002	0.4778	0.4292	0.3300
NAB		–	0.4750	0.4304	0.3257
BHP			–	0.6474	0.3313
RIO				–	0.2959
WOW					–

Table 5.1: Unconditional correlations of 5-minute intraday returns for each pair of stocks, raw returns $r_{t,i}$ used, entire period spans 4 January 2011 to 29 December 2012.

Sample Autocorrelation Functions: Intraday Correlations

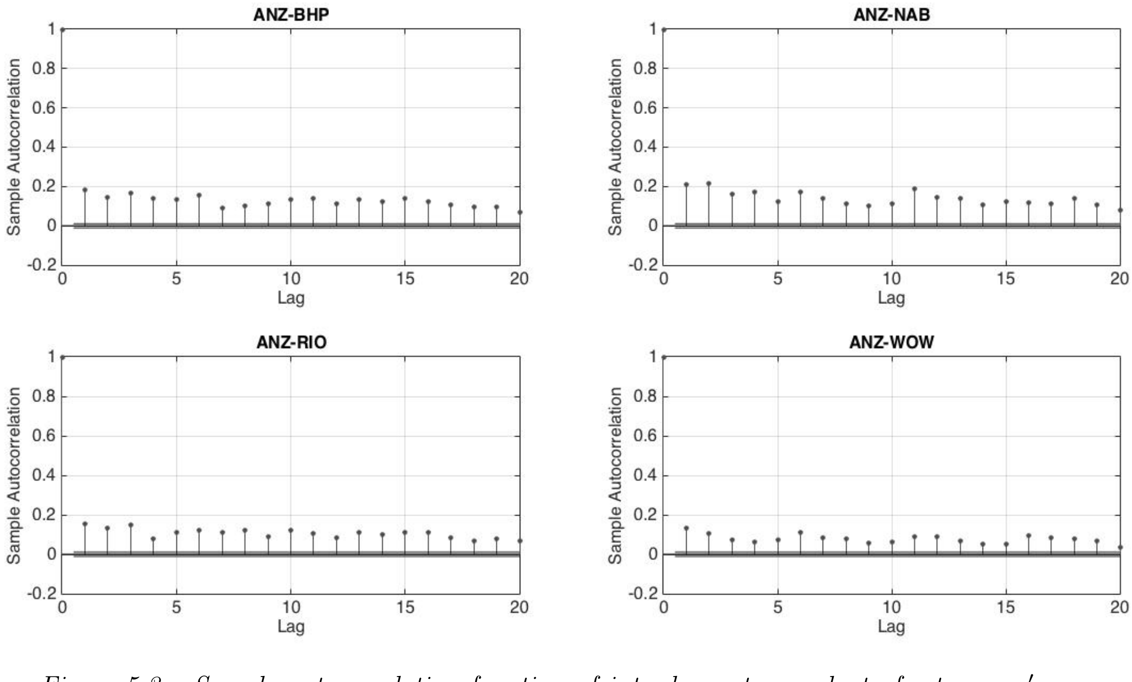

Figure 5.3: Sample autocorrelation function of intraday outer product of returns, $\boldsymbol{r}'_{t,i}\boldsymbol{r}_{t,i}$, for 4 of the 10 pairs. Entire period spans 4 January 2011 to 29 December 2012.

Figures 5.3 and 5.4 contain a selection of the sample autocorrelation functions of the intraday outer product of returns, $\boldsymbol{r}'_{t,i}\boldsymbol{r}_{t,i}$, and daily outer product of returns, $\boldsymbol{r}'_t\boldsymbol{r}_t$ respectively. Certainly persistence is evident both at the 5-minute frequency and at the

Sample Autocorrelation Functions: Daily Correlations

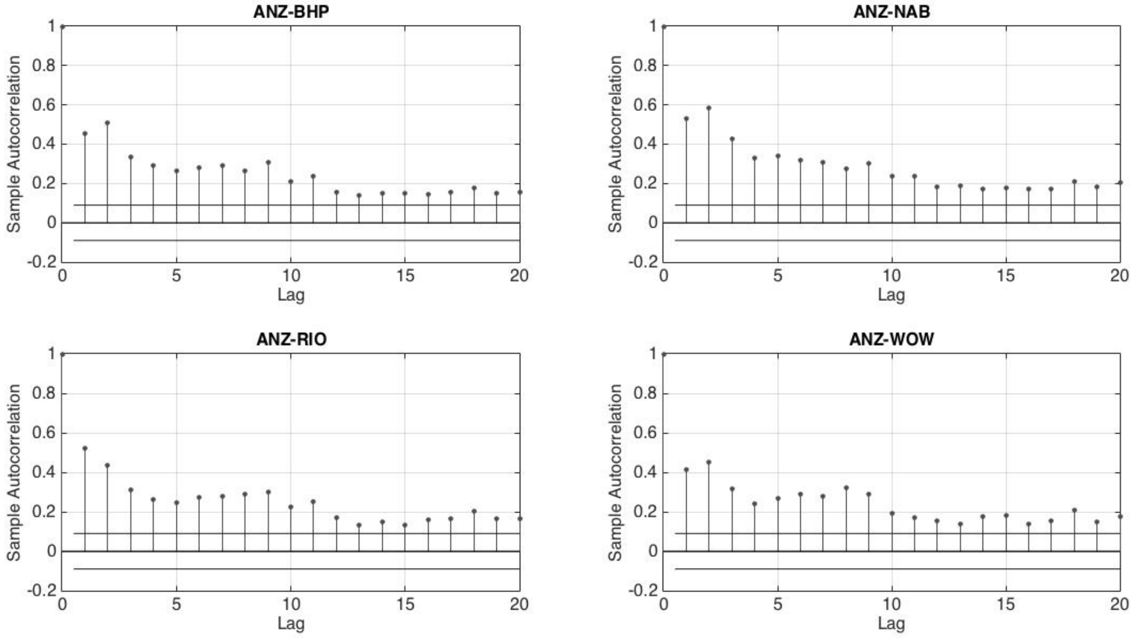

Figure 5.4: Sample autocorrelation function of daily outer product of returns, $\mathbf{r}_t'\mathbf{r}_t$*, for 4 of the 10 pairs. Entire period spans 4 January 2011 to 29 December 2012.*

daily level. Of particular interest in this chapter however, is whether this persistence remains evident in the pairwise relationships after the individual volatilities have been accounted for, and this is where the focus turns now.

5.4 Preliminary Analysis

For the analysis, the returns are volatility standardised, denoted $\hat{\boldsymbol{\epsilon}}_{t,i}$, using the univariate multiplicative component GARCH model outlined in Section 5.2.1. These volatility adjusted returns are shown in Figure 5.5 and it is easily seen that the periods of turbulence and calm have normalised when compared to the raw returns of Figure 5.1. In essence, the intraday volatility adjusted returns are similar to what would be expected of volatility standardised returns at a lower frequency (for example, daily).

It is useful to again consider the unconditional correlations (this time of the volatility standardised returns) and Table 5.2 contains these values. In line with expectations, the unconditional correlations are similar to those in Table 5.1, leading to the same qualitative conclusions described above.

Volatility Standardised Returns

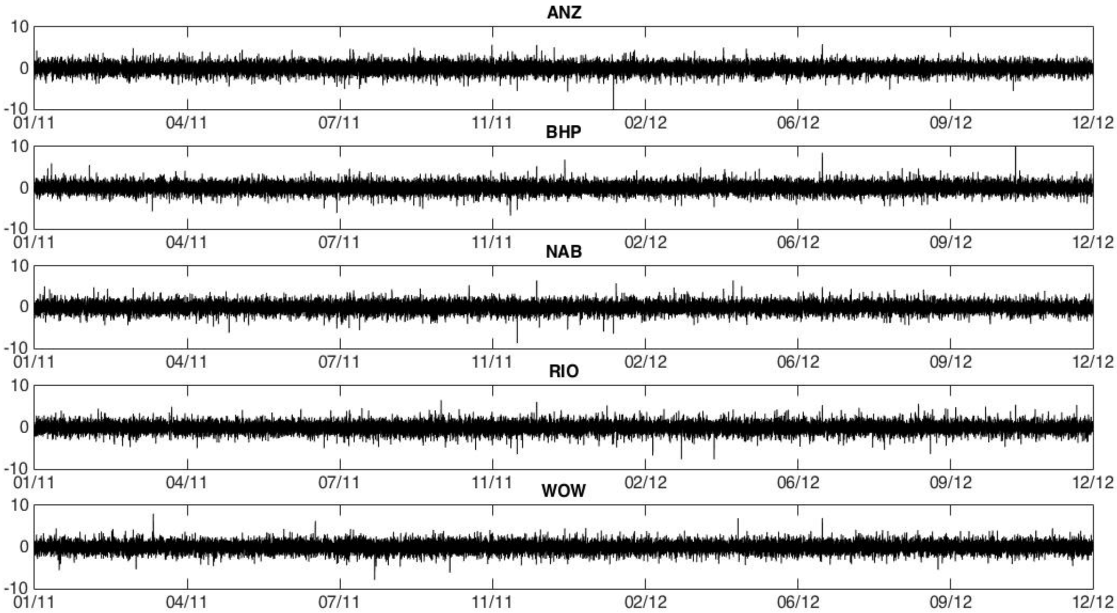

Figure 5.5: Volatility standardised returns, $\hat{\boldsymbol{\epsilon}}_{t,i}$. Entire period spans 4 January 2011 to 29 December 2012.

Unconditional Correlations, volatility adjusted

Industry	Banking		Resources		Retail
ρ	ANZ	NAB	BHP	RIO	WOW
ANZ	–	0.5353	0.4493	0.4066	0.3172
NAB		–	0.4435	0.4026	0.3106
BHP			–	0.6148	0.3264
RIO				–	0.2960
WOW					–

Table 5.2: Unconditional correlations of 5-minute intraday returns for each pair of stocks, volatility adjusted returns $\hat{\boldsymbol{\epsilon}}_{t,i}$ used, entire period spans 4 January 2011 to 29 December 2012.

Figure 5.6 plots the pairwise intraday correlations contained in $\bar{\mathbf{Q}}_i^{DI}$ of equation 5.6. Recall this is the outer product of volatility standardised returns, averaged over the T days of the sample and scaled to be a true correlation matrix. A pattern over the trading day can be seen, as each of the pairwise relationships show an inverted U-shape.

The inverted U-shape is clearly shown when the trading day is broken into sessions, as in Table 5.3, which displays the mean of the pairwise intraday correlations in $\bar{\mathbf{Q}}_i^{DI}$ over three periods of trade. The three sessions are defined as *Morning*, 10:10AM to 11:30AM; *Middle* of the day, 11:30AM to 14:30PM; and, *Afternoon*, 14:30PM to 16:00PM. It is clear

Average Intraday Correlations, volatility adjusted: $\bar{\mathbf{Q}}_i^{DI}$

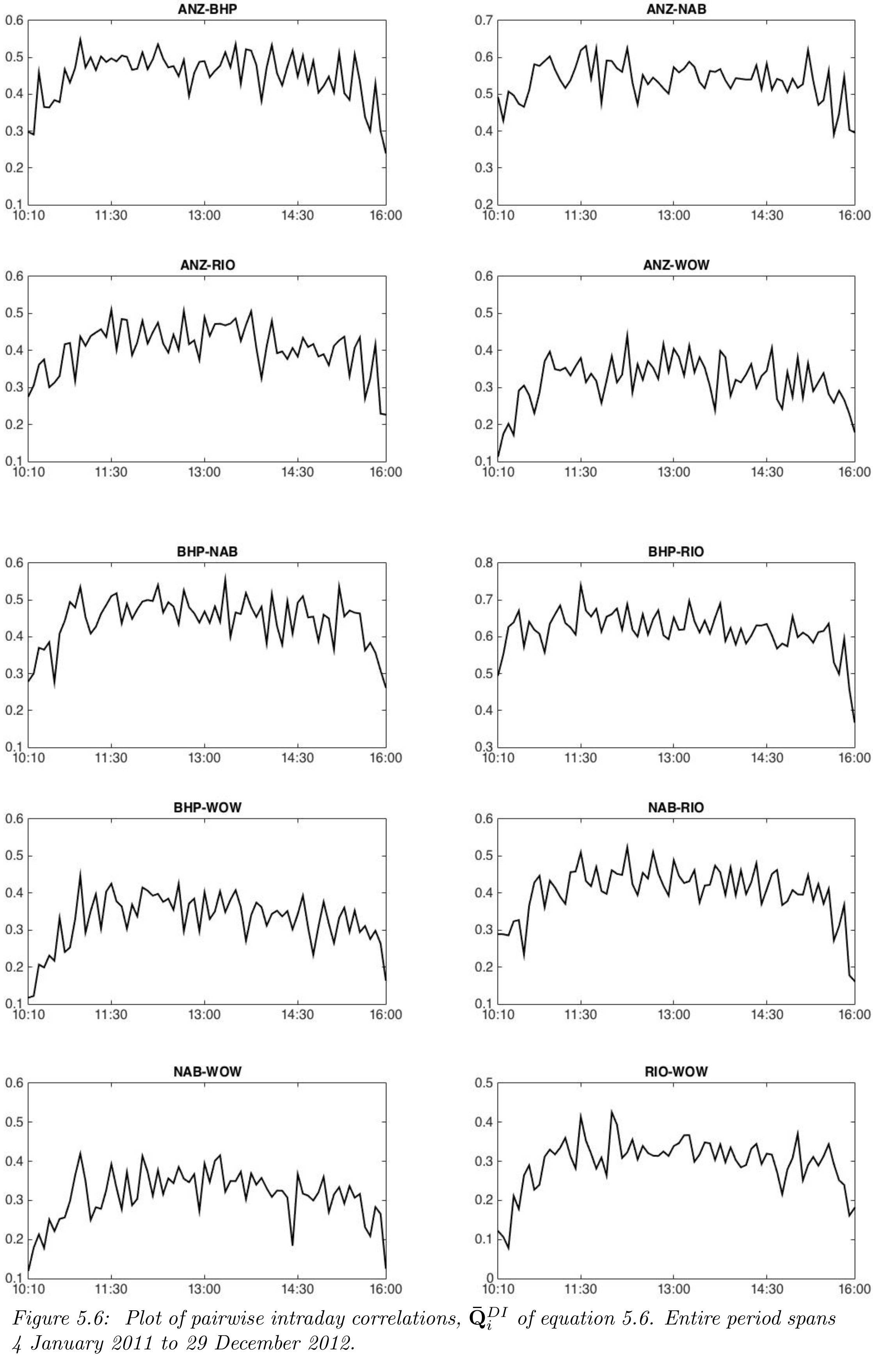

Figure 5.6: Plot of pairwise intraday correlations, $\bar{\mathbf{Q}}_i^{DI}$ *of equation 5.6. Entire period spans 4 January 2011 to 29 December 2012.*

$\bar{\mathbf{Q}}_i^{DI}$: Means over Trading Day

Morning - 10:10AM to 11:30AM					
Mean	ANZ	NAB	BHP	RIO	WOW
ANZ	–	0.5287	0.4304	0.3782	0.2851
NAB		–	0.4110	0.3671	0.2650
BHP			–	0.6146	0.2777
RIO				–	0.2478
WOW					–
Middle - 11:30AM to 14:30PM					
Mean	ANZ	NAB	BHP	RIO	WOW
ANZ	–	0.5570	0.4802	0.4412	0.3482
NAB		–	0.4801	0.4472	0.3463
BHP			–	0.6466	0.3705
RIO				–	0.3332
WOW					–
Afternoon - 14:30PM to 16:00PM					
Mean	ANZ	NAB	BHP	RIO	WOW
ANZ	–	0.5267	0.4435	0.4019	0.3157
NAB		–	0.4409	0.3958	0.3125
BHP			–	0.5987	0.3263
RIO				–	0.2994
WOW					–

Table 5.3: The mean of the pairwise average intraday correlations, $\bar{\mathbf{Q}}_i^{DI}$. Trading day split into three sessions, entire period spans 4 January 2011 to 29 December 2012.

for each pair that the mean value is higher during the middle session, further illustrating the pattern evident in the intraday correlations of Figure 5.6. Possible reasoning for these differences may include increased firm level, or idiosyncratic, effects at the beginning of the day. These idiosyncratic effects are likely due to variations in news arrival between firms before, and soon after, the start of trade.

Interestingly, it appears from Figure 5.6 that the diurnal pattern is strongest for those pairs that are otherwise weakly correlated in this context. For example, the banking pair ANZ and NAB display a very subtle curve that only slightly deviates from their unconditional level of correlation of 0.54. In contrast, the between-industry pairing of RIO (resources) and WOW (retail) reveals a pronounced rise during the morning session of trade, between 10:10AM and 11:30AM. It is worth noting this pair is also the least correlated (unconditionally) of the ten pairs contained in dataset. Indeed, calculating the difference between the mean of the morning session and that of the middle of the day reveals a difference of 0.09 for the RIO-WOW pairing. In comparison, ANZ-NAB has a difference of 0.03. The apparent relationship between the unconditional level of correlations and the difference in means is not as pronounced in the afternoon.

Figure 5.7 contains the daily level pairwise correlations contained in $\bar{\mathbf{Q}}_t^{DY}$ of equation 5.8. It is the outer products of the volatility standardised returns averaged over the I intervals for each of the T days and scaled to be a true correlation matrix. All pairs display similar trends over the sample, although the magnitude of changes in the correlations are larger for some than others.

5.5 Estimation Results

To examine the industry effect on the intraday diurnal pattern further, three portfolios are formed. The first contains 3 stocks of different industries, namely NAB, RIO and WOW; the second contains 4 stocks from two industries; and, the final portfolio contains all 5 equities. This section provides a summary of the estimation results. To begin the analysis, Figures 5.8 to 5.10 show the average portfolio return and average correlation (in the case of DCC, top panel) or equicorrelation (bottom panel) over the entire sample period for each portfolio. The original cDCC and DECO models are used, with the pseudo-correlation $\mathbf{Q}_{t,i}$ shown in equation 5.5. Unsurprisingly, across all portfolio sizes, the correlations increase during the periods of relative market turbulence, approximately March 2011 and June 2011 to December 2011. In 2012, the second half of the dataset, the level of the correlations stabilise somewhat and this supports the idea of mean reversion in the correlations, at least over the time horizon studied here. The average correlation from the cDCC is smoother in comparison to the equicorrelation estimate, this is not unusual for these models and has been noted at the daily frequency in Chapter 4. Despite the difference, the two models are otherwise very similar across the various portfolios.

In terms of differences between the portfolios, certainly the expectation is for the diverse industry portfolio ($N = 3$) to have an overall lower level of correlation than the portfolio of industry pairs ($N = 4$) and this is the case. The $N = 5$ portfolio contains all stocks in the dataset and is roughly an average of the two sub-portfolios in terms of the level of the correlations. This is in line with assumptions of the comparative behaviour of

Daily Correlations, volatility adjusted: $\bar{\mathbf{Q}}_t^{DY}$

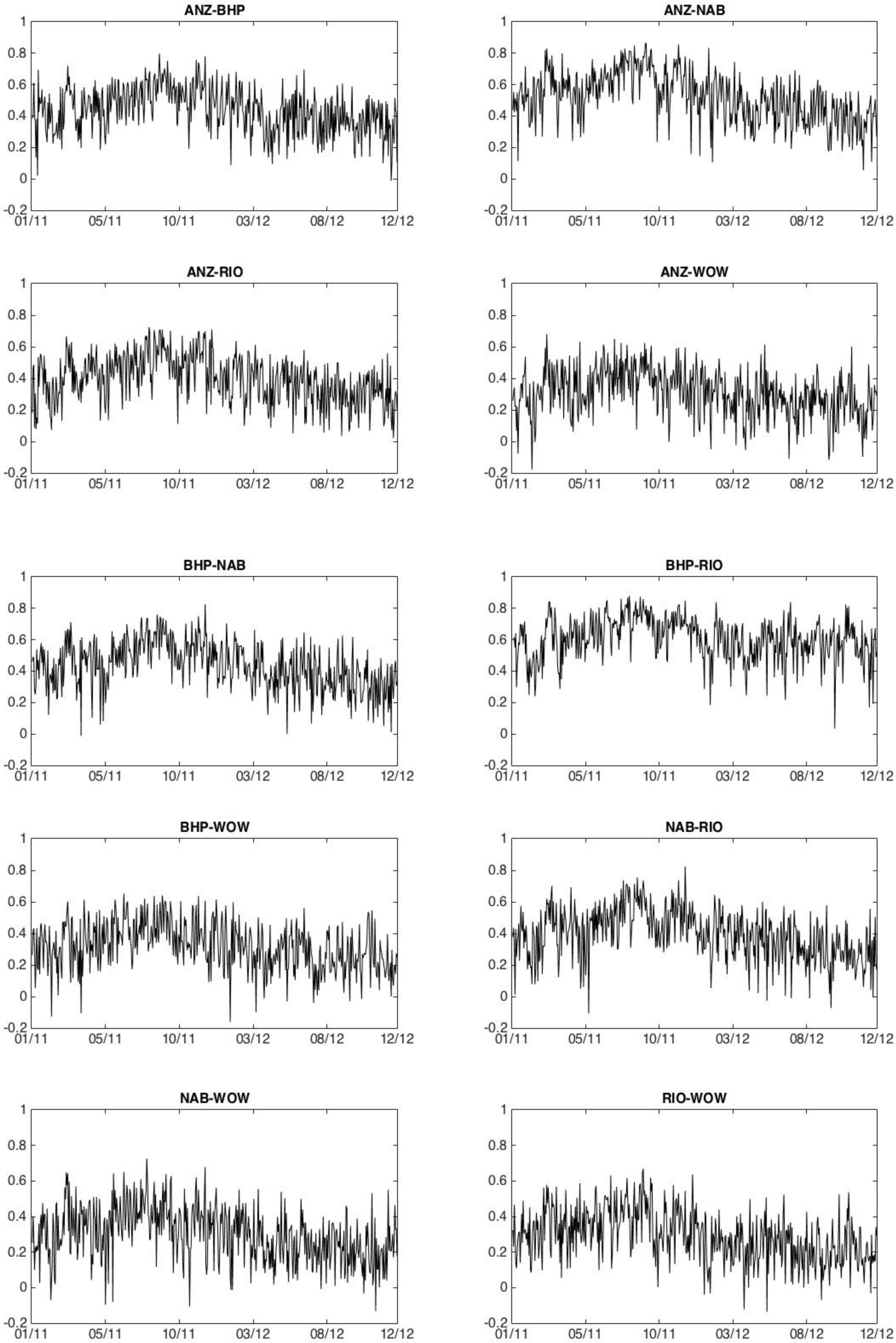

Figure 5.7: Plot of daily pairwise correlations contained in $\bar{\mathbf{Q}}_t^{DY}$ of equation 5.8. Entire period spans 4 January 2011 to 29 December 2012.

the dynamics of this portfolio with the others, given it is a mid-point of the between- and within-industry examples.

$N = 3$, Diverse Industry Portfolio: Average Return & Equicorrelation

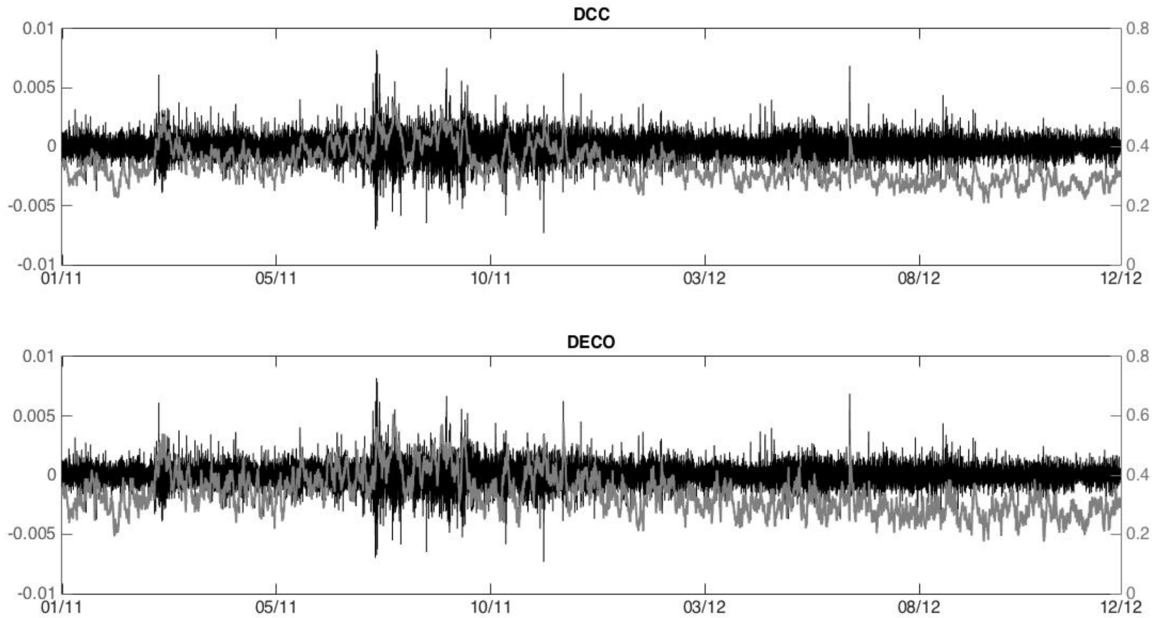

Figure 5.8: Average returns of portfolio of Australian equities and average cDCC correlation or equicorrelation, ρ_t, over entire period, spans 4 January 2011 to 29 December 2012. $N = 3$, Diverse Industry Portfolio: NAB, RIO and WOW.

Parameter estimates for each of the models outlined in Section 5.2 are contained in Tables 5.4 to 5.6, along with log-likelihood and information criterion (IC). There is little difference between the models in terms of log-likelihood and IC, all models estimate easily and appear to fit the data well over the sample. For the diverse 3 stock portfolio in Table 5.4, consisting of NAB, RIO and WOW, the DCC-Both specification provides promising log-likelihoods and IC. This implies it is important to account for both intraday and daily components in the correlations. The same qualitative results are drawn for the $N = 5$ portfolio (see Table 5.6). This is unsurprising as all stocks are included in this case, providing a larger range of unconditional correlation pairings than in the 'Industry Pairs' portfolio, $N = 4$ (Table 5.5). There do appear to be differences between methods in the context of pairs from the same industry, with the equicorrelated models appearing to be further from the cDCC-based models in terms of fit over the sample than for $N = 3$ and $N = 5$. It would be interesting to assess whether this hint of an industry effect translated into significant differences out-of-sample, and this is certainly an avenue for future

$N = 4$, Industry Pairs Portfolio: Average Return & Equicorrelation

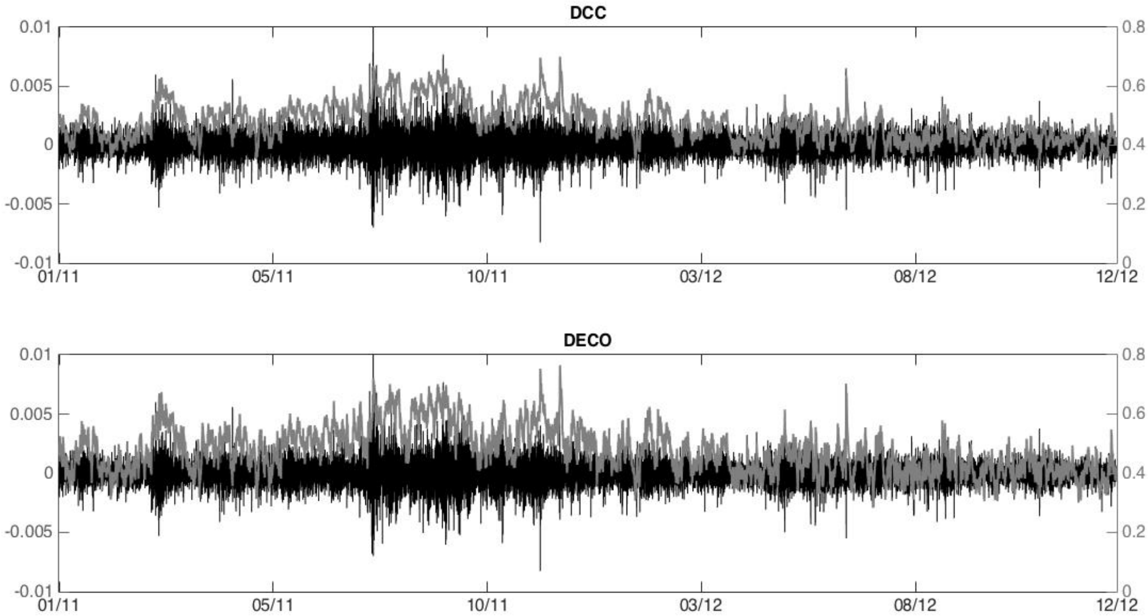

Figure 5.9: Average returns of portfolio of Australian equities and average cDCC correlation or equicorrelation, ρ_t, over entire period, spans 4 January 2011 to 29 December 2012. $N = 4$, Industry Pairs Portfolio: ANZ, NAB, BHP and RIO.

$N = 5$, All Stocks Portfolio: Average Return & Equicorrelation

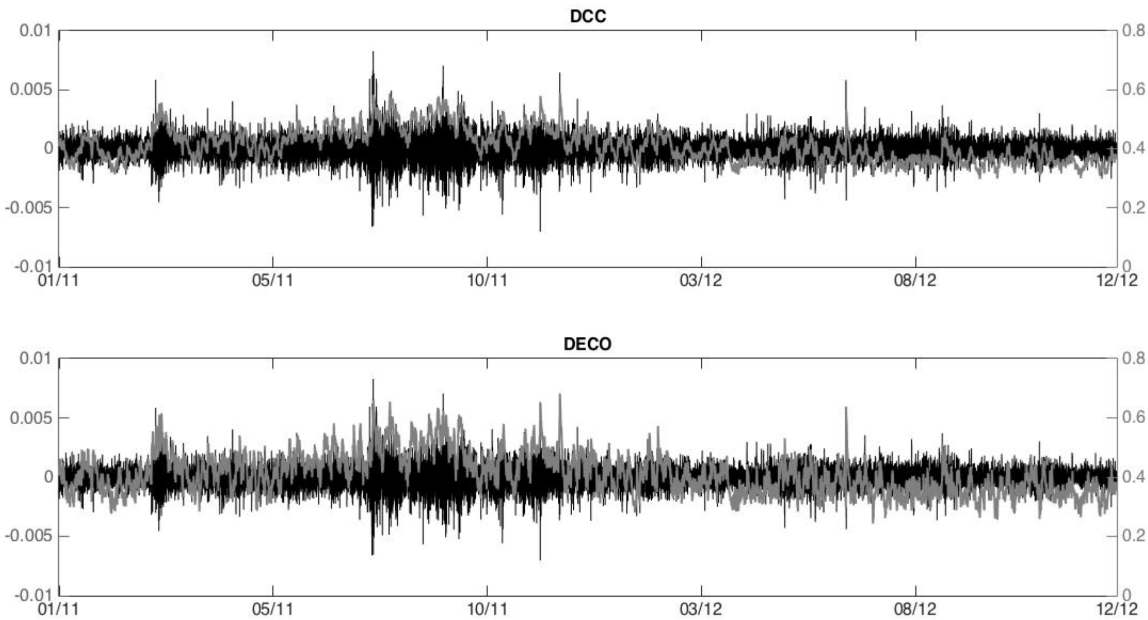

Figure 5.10: Average returns of portfolio of Australian equities and average cDCC correlation or equicorrelation, ρ_t, over entire period, spans 4 January 2011 to 29 December 2012. $N = 5$, All Stocks Portfolio: ANZ, BHP, NAB, RIO and WOW.

$N = 3$, Diverse Industry Portfolio: Full Sample Estimates

Model	a	b	c	Log-Likelihood	AIC	BIC
$N = 3$						
cDCC	0.0060 (0.0018)	0.9886 (0.0043)		515875	-1031717	-1031573
DCC-Intraday	0.0084 (0.0032)	0.0647 (0.0067)		515897	-1031759	-1031616
DCC-Daily I	0.0028 (0.0007)	0.9942 (0.0025)		515739	-1031444	-1031301
DCC-Daily II	0.0086 (0.0067)		0.3083 (0.0225)	515747	-1031459	-1031315
DCC-Daily III	0.0076 (0.0016)	0.9815 (0.0065)	0.0019 (0.0014)	515876	-1031716	-1031564
DCC-Both	0.0059 (0.0010)		0.2504 (0.0178)	**516005**	**-1031976**	**-1031832**
DECO	0.0098 (0.0021)	0.9831 (0.0042)		515859	-1031685	-1031541
DECO-Intraday	0.0165 (0.0077)	0.9825 (0.0086)		515365	-1030695	-1030552
DECO-Daily I	0.0055 (0.2725)	0.9935 (0.4996)		515763	-1031492	-1031348
DECO-Daily II	0.0086 (0.0016)		0.4423 (0.0261)	515719	-1031405	-1031261
DECO-Daily III	0.0128 (0.0084)	0.9688 (0.0416)	0.0057 (0.0163)	515857	-1031678	-1031526
DECO-Both	0.0027 (0.0234)		0.3575 (0.0312)	515985	-1031936	-1031792

Table 5.4: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. Entire period spans 4 January 2011 to 29 December 2012. N = 3, Diverse Industry Portfolio: NAB, RIO and WOW.

$N = 4$, Industry Pairs Portfolio: Full Sample Estimates

Model	a	b	c	Log-Likelihood	AIC	BIC
$N = 4$						
cDCC	0.0095 (0.0010)	0.9830 (0.0022)		750400	-1500757	-1500571
DCC-Intraday	0.0240 (0.0020)	0.1113 (0.0354)		750273	-1500501	-1500315
DCC-Daily I	0.0067 (0.0012)	0.9903 (0.0028)		750022	-1500000	-1499814
DCC-Daily II	0.0225 (0.0020)		0.2605 (0.0131)	749945	-1499846	-1499660
DCC-Daily III	0.0103 (0.0013)	0.9793 (0.0041)	0.0013 (0.0008)	750404	-1500761	-1500567
DCC-Both	0.0206 (0.0017)		0.2195 (0.0119)	**750495**	**-1500947**	**-1500761**
DECO	0.0157 (0.0015)	0.9754 (0.0027)		750325	-1500605	-1500419
DECO-Intraday	0.0222 (0.0062)	0.9768 (0.0070)		749270	-1498497	-1498311
DECO-Daily I	0.0131 (0.0023)	0.9853 (0.0029)		749942	-1499841	-1499655
DECO-Daily II	0.0197 (0.0049)		0.5093 (0.0214)	749670	-1499296	-1499110
DECO-Daily III	0.0183 (0.0015)	0.9640 (0.0039)	0.0056 (0.0016)	750322	-1500598	-1500403
DECO-Both	0.0152 (0.0098)		0.4228 (0.0199)	750303	-1500561	-1500375

Table 5.5: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. Entire period spans 4 January 2011 to 29 December 2012. N = 4, Industry Pairs Portfolio: ANZ, NAB, BHP and RIO.

$N = 5$, All Stocks Portfolio: Full Sample Estimates

Model	a	b	c	Log-Likelihood	AIC	BIC
$N = 5$						
cDCC	0.0069 (0.0008)	0.9867 (0.0020)		979538	-1959023	-1958795
DCC-Intraday	0.0181 (0.0018)	0.0644 (0.0202)		979640	-1959226	-1958998
DCC-Daily I	0.0023 (0.0007)	0.9967 (0.0024)		979194	-1958335	-1958107
DCC-Daily II	0.0174 (0.0016)		0.2686 (0.0117)	979144	-1958234	-1958006
DCC-Daily III	0.0179 (0.0017)	0.0286 (0.0022)	0.2606 (0.0113)	979148	-1958240	-1958003
DCC-Both	0.0160 (0.0018)		0.2167 (0.0112)	**979901**	**-1959747**	**-1959519**
DECO	0.0152 (0.0016)	0.9744 (0.0030)		979365	-1958675	-1958447
DECO-Intraday	0.0181 (0.0031)	0.9674 (0.0072)		979342	-1958630	-1958402
DECO-Daily I	0.0123 (0.0037)	0.9862 (0.0054)		978775	-1957495	-1957267
DECO-Daily II	0.0139 (0.0187)		0.5044 (0.0217)	978875	-1957696	-1957467
DECO-Daily III	0.0207 (0.0045)	0.9478 (0.0218)	0.0117 (0.0093)	979345	-1958634	-1958397
DECO-Both	0.0079 (0.0053)		0.4065 (0.0204)	979702	-1959350	-1959122

Table 5.6: Parameter estimates and robust standard errors; log-likelihood values; and AIC and BIC values. Entire period spans 4 January 2011 to 29 December 2012. $N = 5$, All Stocks Portfolio: ANZ, BHP, NAB, RIO and WOW.

research. A possible explanation for this contrasting behaviour between the diverse- and within-industry examples relates to the ability of the cDCC-based models to capture the pairwise dynamics more effectively than the corresponding DECO specifications when a larger proportion of pairs in the portfolio are highly correlated.[3] As was discussed in the previous section, the pairings exhibiting higher unconditional correlations displayed a less pronounced intraday diurnal pattern in the correlation structure than less correlated pairs. These differences appear important in this case.

Worth highlighting is the large difference in the estimated b coefficient, the parameter weighting of the previous interval's pseudo-correlation, $\mathbf{Q}_{t,i-1}$, in the context of the '-Intraday' models specifically. For the cDCC-based models, this parameter is much lower. For example, in $N = 3$ the DCC-Intraday $b = 0.07$ compared to 0.98 for DECO-Intraday. This leads to the overall persistence in the correlations, that is $a + b$, to be much lower for the DCC-Intraday model than DECO-Intraday, implying the contribution of present and past information in the correlation dynamics is very different. The impact of this difference in terms of the behaviour of correlations is most easily displayed visually, as in Figures 5.11

[3]DCC's ability to capture the dynamics of highly correlated series' was noted in Section 4.5 of Chapter 4, in a daily setting.

Average DCC-Intraday Correlations

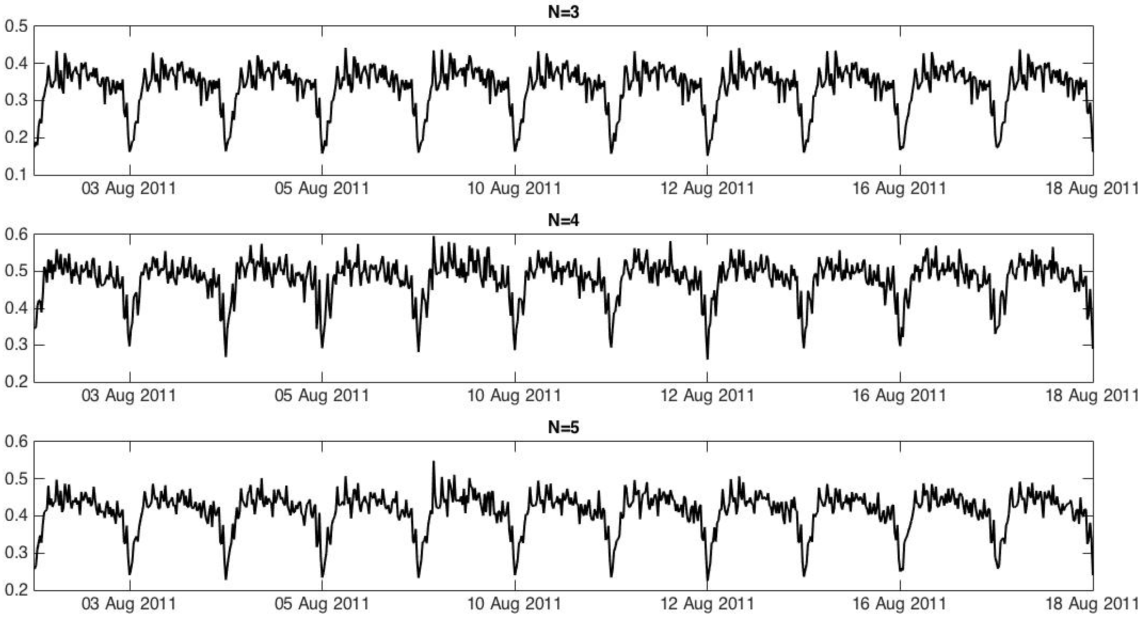

Figure 5.11: Average DCC-Intraday correlations over the period 2 August 2011 to 18 August 2011, for each portfolio.

DECO-Intraday Equicorrelations

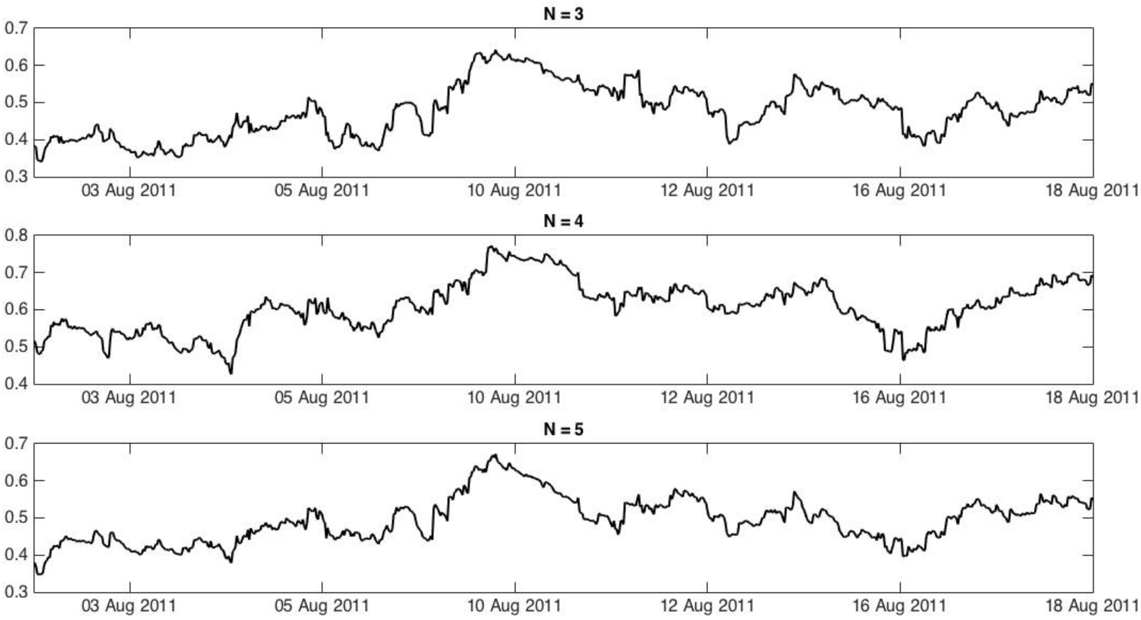

Figure 5.12: DECO-Intraday equicorrelations over the period 2 August 2011 to 18 August 2011, for each portfolio.

and 5.12. For ease of interpretation, three trading weeks have been presented, beginning 2 August 2011 to 18 August 2011. This particular period was the most volatile of the entire dataset, corresponding to the downgrading of US credit. For the DCC-Intraday models across all portfolios, the impact of the estimated b parameter is clear. A much heavier weight is given to the diurnal pattern over the trading day, with the correlations reverting readily to the intercept of $\bar{\mathbf{Q}}_i^{DI}$. This difference may be important in a practical sense and forecasting exercises would shed light on these dynamics out-of-sample. It is conjectured that in cases where the level of pairwise unconditional correlations is more diverse, like that in the Diverse Industry and All Stocks portfolios, accounting for this inverted U-shape in the correlations over the trading day is beneficial. Consideration of the respective parameter values across the three portfolios for the DCC-Intraday model reveals higher estimated coefficients a and b for $N = 4$. This implies correlations are more persistent for the 'Industry Pairs' portfolio than the more diverse portfolios.

As mentioned above, for the 'Diverse' and 'All Stocks' portfolios, capturing the intraday diurnality in the correlations appears to be as important as capturing temporal dependence at the daily frequency. This is reflected in the similar log-likelihood and IC values of the DCC-Intraday model in comparison to the DCC-Daily models, for both $N = 3$ and 5 portfolio sizes. Of the DCC-Daily models, DCC-Daily III provides a reasonable fit and it is the same in the case of the equivalent DECO-Daily model group. This is presumably due to the fact that the Daily III models are an unrestricted version, aiming to account for daily level persistence additively whilst allowing for both present and past information explicitly. This seems to indicate that a complete picture of correlation persistence is helpful in the absence of directly capturing the diurnal intraday pattern.

Of course, the isolation of a three week trading period also provides a useful illustration of how the '-Both' models behave. Figures 5.13 and 5.14 show the dynamics of the correlations for the DCC-Both and DECO-Both models over the period of 2 August 2011 to 18 August 2011. Unlike the differences evident in the '-Intraday' example, both the non-equicorrelated and equicorrelated versions display a clear pattern over the trading day. Recall that these models are designed to capture both the intraday pattern in correlations and correlation clustering at the daily frequency. In terms of parameters, the estimated coefficient a, which governs the input of new information into the pseudo-correlation, is

Average DCC-Both Correlations

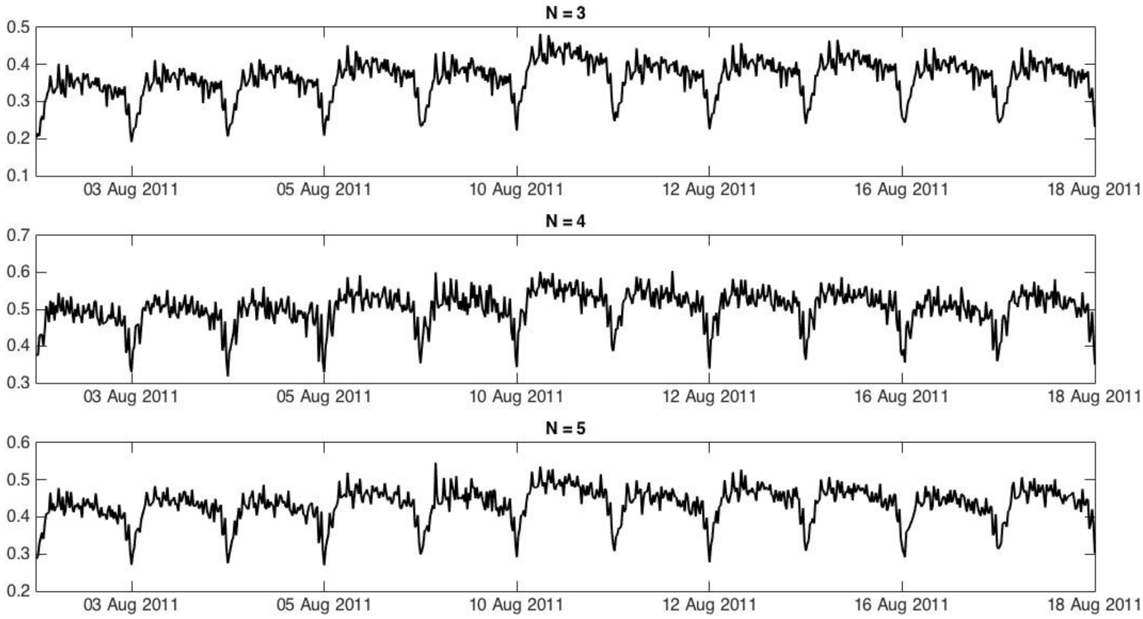

Figure 5.13: Average DCC-Both correlations over the period 2 August 2011 to 18 August 2011, for each portfolio.

DECO-Both Equicorrelations

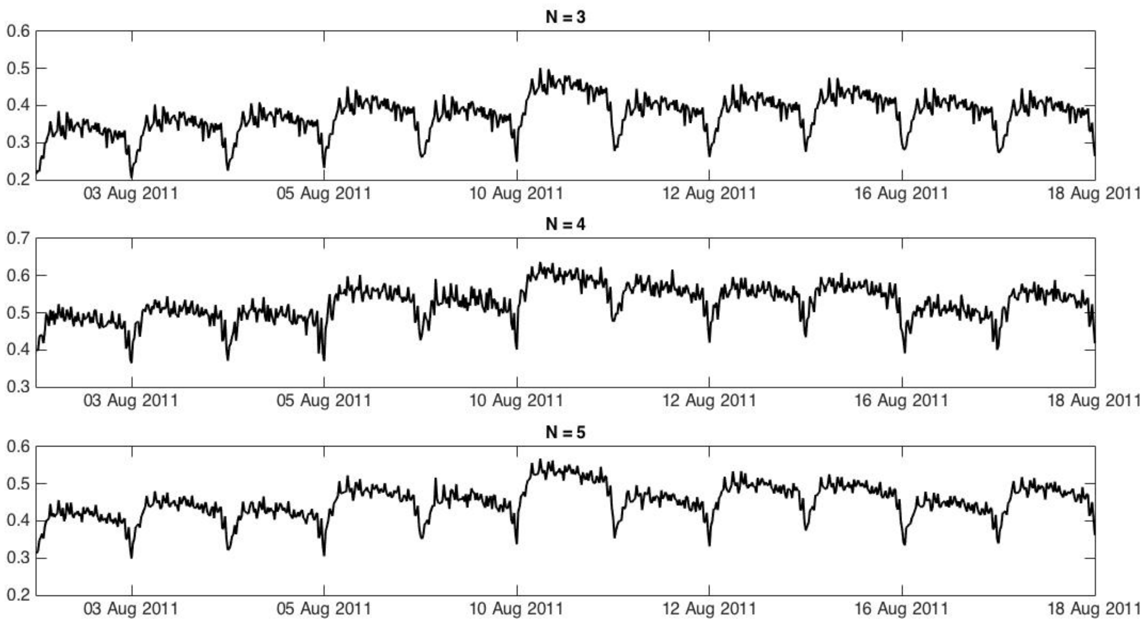

Figure 5.14: DECO-Both equicorrelations over the period 2 August 2011 to 18 August 2011, for each portfolio.

similar to the other models. Recall that the '-Both' specification omits the lagged pseudo-correlation $\mathbf{Q}_{t,i-1}$, thus there is no b coefficient to report. Instead, each '-Both' model is designed to capture the daily level persistence in correlations through the additive term $\bar{\mathbf{Q}}_t^{DY}$, with the coefficient c. In general, the DCC-Both model has lower estimated values of c than the corresponding DECO-Both model. This effectively smooths the equicorrelated process over the day, as the daily level correlation is given a higher weighting.

5.6 Conclusion

The availability of high frequency intraday data has presented opportunities to model the intraday correlation dynamics of a portfolio of assets. Modelling of these processes is important for a range of practical applications over the trading day, including hedging, risk management, trade scheduling and setting limit orders. Traders and market makers require up-to-date information at increasingly small intervals, motivating studies such as that contained in this chapter. Increased reliance on computerised trading depends on forecasts of volatility and as financial decisions are realistically made in terms of more than one asset, correlations are important. Given a thorough search of the intraday correlation literature, this is the first piece of research to explicitly consider modelling high frequency intraday correlation dynamics and the first to use the MGARCH framework in this context. Despite the interest in modelling intraday volatilities of individual assets, the area of intraday correlations is a relatively new literature.

This chapter outlined key features of the behaviour of correlations over the trading day. In particular, evidence of an inverted U-shape pattern in the intraday correlations is found. Further, several models based on Dynamic Conditional Correlation and Dynamic Equicorrelation are presented to capture this intraday diurnal pattern. Results of full sample estimation indicate that it may be worthwhile to incorporate both day-to-day correlation persistence and the intraday pattern in the correlations. In terms of the intraday pattern evident in the correlations, stocks that are highly correlated such as those from the same industry, seem to remain highly correlated over the course of the trading day and thus display a correlation diurnality that is less pronounced. In the context of a portfolio of highly correlated stocks, the cDCC family of models capture the correlation dynamics better than the corresponding equicorrelated versions. More generally, the cDCC framework appears to model the intraday pattern more effectively than DECO.

The insights into the behaviour of equity correlations at a high intraday frequency outlined here and the novel modelling approach suggested to capture these patterns contribute to the intraday correlation literature. Future research could consider alternative models for capturing the intraday pattern in correlations. For example, a seasonal ARIMA (see Hamilton, 1994, for a thorough examination of this class of model) or the smooth transition GARCH framework of Silvennoinen and Teräsvirta (2015) may be appropriate.

There are numerous extensions possible for this work, including the application of the models to larger portfolio sizes and forecasting. The former pertains to the size of the portfolio used in the chapter. Five stocks, although a useful sample size for performing a range of analyses and drawing interesting inferences, is a small number. Future research could see these intraday correlation models applied to a larger number of stocks. Forecasting, a natural and indeed practical extension to the work presented in this chapter, promises to yield interesting results given the findings detailed here. Unlike previous chapters of the thesis focusing on daily returns, the evaluation methodology used and portfolio allocations example is not of benefit here. It is unlikely simple rebalancing of a portfolio would occur at such high frequencies. Rather, it is logical to undertake a different approach to compare the intraday correlation forecasting models. A host of financial applications require intraday measures of risk and such techniques would benefit derivative traders and institutional investors, for example hedge funds. Future work in terms of forecasting exercises could be along these lines.

Chapter 6

Conclusion

Many financial decisions are dependent on expectations of the volatility of asset returns. To form an accurate picture of the risk of a portfolio, an expectation of the correlation between the assets is required. Given the importance of volatility and correlation of asset returns to finance, generating accurate forecasts is crucial. The research in this dissertation contributes to the extensive correlation modelling literature in four ways. Firstly, it provided insights into the optimal use of correlation forecasting models in the context of large portfolios. Secondly, conditioning the correlation structure of a portfolio of securities on market volatility generally led to an improvement in portfolio outcomes. Contrasting results between two empirical examples suggested the benefits of equicorrelation are tempered when forecasting correlations between indices rather than equities. Thirdly, the complexities of intraday correlation dynamics were investigated and characteristics unique to these high frequency processes were outlined. Lastly, a novel approach for modelling intraday correlation dynamics was presented and promising results pose interesting questions for future work. This section will elaborate on each of these contributions, highlight limitations of the empirical work and suggest avenues for further research in modelling the correlations of financial asset returns.

Chapter 3 sought to address the open question of how to handle large dimensional correlation matrices. It asked whether simple moving average based models generated adequate forecasts in this scenario relative to more complex correlation forecasting methods. A thorough examination of multivariate GARCH (MGARCH) and in particular, equicorrelation, led to a number of results worth highlighting. Findings indicated that moving average style models were outperformed in a portfolio allocation setting by the MGARCH models used, however in certain circumstances more basic MGARCH models were sufficient. For example, equicorrelation was found to be useful during periods of market turbulence, such as crises, while a simple Constant Conditional Correlation framework could not be discounted for periods of tranquility. Further, the success of assuming equicorrelation in the large dimensional portfolio setting provided motivation for exploring possible extensions to this modelling framework.

In Chapter 4, linking volatility to correlations by conditioning the equicorrelation structure on volatility was suggested as an improvement to the existing Dynamic Equicorrelation framework. Performance of the resulting models was evaluated through two empirical examples, with both a national and international setting studied. Significant insights into the behaviour of the MGARCH family of models and equicorrelation in particular were gained, deepening the understanding of these techniques for generating forecasts of large correlation matrices and the scenarios under which these models perform optimally. Overall, the volatility dependent structures led to improved portfolio outcomes.

The thesis also examined the intraday correlation dynamics of returns sampled at high frequency. It provided insights into how to account for the unique features of the high frequency correlation process (Chapter 5). The inverted U-shape of the intraday correlations between assets was identified. Further, this intraday pattern is most evident between stocks that have a lower level of unconditional correlation, such as those from different industries. A novel MGARCH approach was developed by adapting Dynamic Conditional Correlation and Dynamic Equicorrelation models to account for both daily-level fluctuations in the correlations and the intraday pattern evident in the correlations during the trading day. It was found that capturing both effects may be advantageous and provided a reasonable fit over the sample studied. The research into capturing intraday correlation dynamics leads to many interesting questions, many of which remain open in the literature.

As is the nature of empirical research, there are some limitations of the studies contained in this thesis. Some of these lead to ideas for future research. In the first instance, results of the empirical applications documented here could be sample-specific, that is affected by the time series analysed. Certainly, it would be of interest to replicate the methodologies outlined in this thesis for portfolios of different stocks over the same time period; further, replicate the studies for different time periods and length of time series samples. To do so would provide information as to the robustness of the findings of this thesis. The second limitation pertains to the portfolio used in Chapter 5 specifically. In the first instance, five stocks is a small portfolio and future research could see the models presented for modelling intraday correlations applied to a larger number of stocks. Secondly, it may be informative to replicate the study using equities of other markets, such as the U.S. or Europe, to assess the similarities (or differences) of the intraday correlation dynamics seen in the Australian portfolio.

With regard to possible research avenues opened by the findings of this dissertation, in addition to those discussed above, the area of modelling intraday correlations promises to yield interesting results. This field is relatively new to the financial econometrics literature and many practical and relevant questions remain open. Understanding how to best model and indeed forecast correlations at high frequencies is of benefit not only from an econometric viewpoint but is relevant to a host of financial applications. It is reasonable to expect that generating risk profiles for large portfolios at high frequencies will be increasingly relevant for traders and market makers.

Appendix A

Chapter 3 Supplement

U.S. Equities: Portfolios

$N = 5$									
AA	ABT	ADM	AEP	AET					
$N = 10$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
$N = 25$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC					
$N = 50$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC	CSX	D	DD	DOV	DOW
DOW	DTE	DUK	ED	EMR	ETN	ETR	EXC	GCI	GD
GE	GIS	GLW	GPC	GPS	GWW	HAL	HNZ	HON	HRB
$N = 100$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC	CSX	D	DD	DOV	DOW
DTE	DUK	ED	EMR	ETN	ETR	EXC	GCI	GD	GE
GIS	GLW	GPC	GPS	GWW	HAL	HNZ	HON	HRB	HSY
IFF	IP	ITT	ITW	JCI	JCP	JPM	K	KMB	KR
LLY	LMT	LNC	MAS	MCD	MDP	MDT	MHP	MRK	NEM
NOC	NSC	PBI	PCG	PEG	PEP	PFE	PG	PH	PHM
PPG	R	RDC	RTN	SLB	SNA	SVU	SWK	T	THC
TJX	TXT	UTX	VFC	WBA	WFC	WHR	WMB	WMT	WY

Table A.1: List of stocks included in each portfolio for U.S. equities dataset.

Details of dataset, including summary statistics [-10pt]

Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
AA	Alcoa Inc.	Materials	-0.175	0.209	0.000	0.027	-0.091	9.934
ABT	Abbott Laboratories	Health Care	-0.176	0.218	0.000	0.017	0.242	15.850
ADM	Archer-Daniels-Midland Co.	Consumer Staples	-0.184	0.160	0.000	0.021	-0.160	11.664
AEP	American Electric Power	Utilities	-0.161	0.181	0.000	0.016	0.262	17.302
AET	Aetna Inc.	Health Care	-0.227	0.254	0.000	0.024	-0.485	15.446
AIG	American Intl Group	Financials	-0.460	0.460	0.000	0.042	-0.505	46.934
AMD	Advanced Micro Devices	Information Technology	-0.392	0.232	0.000	0.042	-0.420	10.247
APD	Air Products & Chemicals Inc.	Materials	-0.131	0.137	0.000	0.019	-0.098	7.653
ATI	Allegheny Technologies Inc.	Materials	-0.213	0.229	0.000	0.033	-0.112	6.859
AVP	Avon Products Inc.	Consumer Staples	-0.324	0.176	0.000	0.023	-0.744	20.053
AXP	American Express Co.	Financials	-0.194	0.188	0.000	0.025	0.008	10.333
BA	The Boeing Co.	Industrials	-0.194	0.144	0.000	0.021	-0.363	9.431
BAX	Baxter International Inc.	Health Care	-0.186	0.104	0.000	0.018	-0.950	13.021
BCR	CR Bard Inc.	Health Care	-0.124	0.198	0.000	0.016	0.338	13.156
BDX	Becton Dickinson & Co.	Health Care	-0.252	0.158	0.000	0.018	-0.584	18.491
BLL	Ball Corp.	Materials	-0.108	0.110	0.001	0.019	0.234	7.497
CAG	Conagra Foods Inc.	Consumer Staples	-0.217	0.104	0.000	0.016	-0.739	17.184
CAT	Caterpillar Inc.	Industrials	-0.157	0.137	0.001	0.022	-0.097	6.712
CB	Chubb Corp.	Financials	-0.134	0.155	0.000	0.019	0.398	10.517
CI	Cigna Corp.	Health Care	-0.247	0.211	0.000	0.024	-0.654	17.632
CL	Colgate-Palmolive Co.	Consumer Staples	-0.173	0.182	0.000	0.016	-0.005	14.397
CLX	Clorox Company	Consumer Staples	-0.176	0.124	0.000	0.017	-0.422	12.933
COP	Conocophillips	Energy	-0.149	0.154	0.000	0.019	-0.297	8.755
CPB	Campbell Soup Co.	Consumer Staples	-0.144	0.183	0.000	0.016	0.369	14.166
CSC	Computer Sciences Corp.	Information Technology	-0.254	0.170	0.000	0.025	-0.492	12.547
CSX	CSX Corp.	Industrials	-0.176	0.141	0.000	0.022	-0.104	7.224
D	Dominion Resources Inc./VA	Utilities	-0.137	0.100	0.000	0.014	-0.584	13.138
DD	DU Pont (E.I.) de Nemours	Materials	-0.120	0.109	0.000	0.020	-0.155	6.954
DOV	Dover Corp.	Industrials	-0.178	0.146	0.000	0.020	-0.112	7.786
DOW	The Dow Chemical Co.	Materials	-0.211	0.169	0.000	0.023	-0.239	9.510
DTE	DTE Energy Co.	Utilities	-0.111	0.122	0.000	0.014	0.045	10.564
DUK	Duke Energy Corp.	Utilities	-0.161	0.150	0.000	0.016	-0.184	13.678
ED	Consolidated Edison Inc.	Utilities	-0.070	0.090	0.000	0.012	0.149	7.684

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Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
EMR	Emerson Electric Co.	Industrials	-0.163	0.143	0.000	0.019	-0.034	8.865
ETN	Eaton Corp. PLC	Industrials	-0.165	0.174	0.000	0.019	-0.090	9.030
ETR	Entergy Corp.	Utilities	-0.196	0.133	0.000	0.016	-0.396	15.075
EXC	Exelon Corp.	Utilities	-0.125	0.159	0.000	0.017	-0.019	10.992
GCI	Gannett Co. Inc.	Consumer Discretionary	-0.274	0.332	0.000	0.025	0.119	23.921
GD	General Dynamics Corp.	Industrials	-0.132	0.111	0.000	0.017	-0.221	7.139
GE	General Electric Co.	Industrials	-0.137	0.180	0.000	0.020	0.019	9.913
GIS	General Mills Inc.	Consumer Staples	-0.119	0.090	0.000	0.012	-0.415	11.185
GLW	Corning Inc.	Information Technology	-0.434	0.196	0.000	0.034	-0.618	14.424
GPC	Genuine Parts Co.	Consumer Discretionary	-0.095	0.096	0.000	0.014	0.171	7.237
GPS	The Gap Inc.	Consumer Discretionary	-0.236	0.241	0.000	0.027	-0.308	11.318
GWW	WW Grainger Inc.	Industrials	-0.147	0.159	0.000	0.018	0.153	8.994
HAL	Halliburton Co.	Energy	-0.303	0.213	0.000	0.030	-0.276	10.140
HNZ	HJ Heinz Co.	Consumer Staples	-0.088	0.103	0.000	0.014	0.134	8.181
HON	Honeywell International Inc.	Industrials	-0.196	0.254	0.000	0.022	-0.254	13.915
HRB	H&R Block Inc.	Consumer Discretionary	-0.197	0.171	0.000	0.022	-0.436	10.830
HSY	The Hershey Co.	Consumer Staples	-0.128	0.225	0.000	0.016	0.734	18.729
IFF	Intl Flavors & Fragrances	Materials	-0.174	0.149	0.000	0.017	-0.316	12.164
IP	International Paper Co.	Materials	-0.205	0.198	0.000	0.024	0.056	10.171
ITT	ITT Corp.	Industrials	-0.117	0.218	0.000	0.019	0.545	12.309
ITW	Illinois Tool Works	Industrials	-0.101	0.123	0.000	0.018	0.098	6.710
JCI	Johnson Controls Inc.	Consumer Discretionary	-0.133	0.128	0.000	0.021	-0.020	7.630
JCP	J.C. Penney Co. Inc.	Consumer Discretionary	-0.221	0.172	0.000	0.027	0.157	7.764
JPM	JP Morgan Chase & Co.	Financials	-0.232	0.224	0.000	0.027	0.209	13.335
K	Kellogg Co.	Consumer Staples	-0.101	0.103	0.000	0.015	0.119	8.745
KMB	Kimberlyark Corp.	Consumer Staples	-0.120	0.101	0.000	0.015	-0.254	10.341
KR	Kroger Co.	Consumer Staples	-0.295	0.097	0.000	0.020	-1.005	18.528
LLY	Eli Lilly & Co.	Health Care	-0.193	0.163	0.000	0.019	-0.157	10.539
LMT	Lockheed Martin Corp.	Industrials	-0.148	0.137	0.000	0.018	-0.190	9.908
LNC	Lincoln National Corp.	Financials	-0.509	0.362	0.000	0.034	-1.197	46.658
MAS	Masco Corp.	Industrials	-0.174	0.168	0.000	0.025	-0.161	8.364
MCD	McDonald's Corp.	Consumer Discretionary	-0.137	0.103	0.000	0.017	-0.090	8.076
MDP	Meredith Corp.	Consumer Discretionary	-0.148	0.152	0.000	0.020	0.081	10.077

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Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
MDT	Medtronic Inc.	Health Care	-0.142	0.112	0.000	0.019	-0.191	8.364
MHP	McGraw-Hill Companies Inc.	Financials	-0.142	0.214	0.000	0.020	0.275	12.265
MRK	Merck & Co. Inc.	Health Care	-0.194	0.231	0.000	0.019	-0.088	15.917
NEM	Newmont Mining Corp.	Materials	-0.187	0.225	0.000	0.028	0.448	8.378
NOC	Northrop Grumman Corp.	Industrials	-0.158	0.213	0.000	0.017	0.154	15.011
NSC	Norfolk Southern Corp.	Industrials	-0.138	0.143	0.000	0.022	-0.016	6.373
PBI	Pitney Bowes Inc.	Industrials	-0.190	0.208	0.000	0.019	-0.539	16.068
PCG	P G & E Corp.	Utilities	-0.224	0.269	0.000	0.021	0.391	32.840
PEG	Public Service Enterprise Gp	Utilities	-0.110	0.158	0.000	0.016	0.063	11.432
PEP	Pepsico Inc.	Consumer Staples	-0.127	0.156	0.000	0.016	0.242	12.193
PFE	Pfizer Inc.	Health Care	-0.118	0.097	0.000	0.018	-0.213	6.647
PG	The Proctor & Gamble Co.	Consumer Staples	-0.159	0.097	0.000	0.015	-0.519	11.502
PH	Parker Hannifin Corp.	Industrials	-0.126	0.128	0.000	0.022	-0.076	6.595
PHM	Pultegroup Inc.	Consumer Discretionary	-0.204	0.207	0.000	0.031	0.125	6.855
PPG	PPG Industries Inc.	Materials	-0.122	0.138	0.000	0.019	0.124	7.581
R	Ryder System Inc.	Industrials	-0.198	0.125	0.000	0.023	-0.388	8.686
RDC	Rowan Companies PLC	Energy	-0.217	0.224	0.000	0.032	-0.119	6.170
RTN	Raytheon Company	Industrials	-0.215	0.237	0.000	0.020	-0.179	19.127
SLB	Schlumberger Ltd	Energy	-0.203	0.139	0.000	0.025	-0.275	7.351
SNA	Snap-On Inc.	Industrials	-0.175	0.141	0.000	0.019	0.027	10.734
SVU	Supervalu Inc.	Consumer Staples	-0.274	0.383	0.000	0.026	-0.447	27.708
SWK	Stanley Black & Decker Inc.	Industrials	-0.146	0.140	0.000	0.021	0.191	7.442
T	AT & T Inc.	Telecommunication Services	-0.135	0.151	0.000	0.018	0.091	7.690
THC	Tenet Healthcare Corp.	Health Care	-0.335	0.439	0.000	0.032	-0.087	29.584
TJX	TJX Companies Inc.	Consumer Discretionary	-0.174	0.163	0.001	0.023	0.059	7.930
TXT	Textron Inc.	Industrials	-0.381	0.398	0.000	0.027	-0.751	33.186
UTX	United Technologies Corp.	Industrials	-0.187	0.128	0.001	0.018	-0.209	9.150
VFC	VF Corp.	Consumer Discretionary	-0.146	0.136	0.001	0.019	0.186	7.945
WBA	Walgreen Co.	Consumer Staples	-0.162	0.155	0.000	0.019	-0.027	8.703
WFC	Wells Fargo & Co.	Financials	-0.272	0.283	0.000	0.026	0.768	25.120
WHR	Whirlpool Corp.	Consumer Discretionary	-0.155	0.176	0.000	0.024	0.150	7.667
WMB	Williams Cos Inc.	Energy	-0.373	0.373	0.000	0.034	-0.802	28.030
WMT	Wal-Mart Stores Inc.	Consumer Staples	-0.107	0.109	0.000	0.018	0.141	7.136

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Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
WY	Weyerhaeuser Co.	Financials	-0.188	0.131	0.000	0.022	-0.145	7.312

Table A.2: The 100 stocks included in the full dataset, including ticker code, company name and sector as well as summary statistics for entire period spans 3 January 1996 to 31 December 2012.

Appendix B

Chapter 4 Supplement

US Equities: Additional Results

US Equities: Full sample log-likelihoods

Model	N	Log-Like	Model	N	Log-Like	Model	N	Log-Like
CCC	5	49615	CCC	10	109079	CCC	25	295974
DCC		56931	DCC		108863	DCC		291432
DCC-ARE		56935	DCC-ARE		108919	DCC-ARE		291291
DCC-AVE		56934	DCC-AVE		108900	DCC-AVE		291417
DCC-TVR		56934	DCC-TVR		108851	DCC-TVR		290066
DCC-TVV		56934	DCC-TVV		108842	DCC-TVV		290067
DEC		56919	DEC		108315	DEC		288704
DEC-ARE		56919	DEC-ARE		108316	DEC-ARE		288707
DEC-AVE		56923	DEC-AVE		108316	DEC-AVE		288759
DEC-TVR		56902	DEC-TVR		108316	DEC-TVR		288676
DEC-TVV		56902	DEC-TVV		108316	DEC-TVV		288676
CCC	50	587112	CCC	100	1172845			
DCC		609913	DCC		1218272			
DCC-ARE		610043	DCC-ARE		1218310			
DCC-AVE		609986	DCC-AVE		1218596			
DCC-TVR		603252	DCC-TVR		1193714			
DCC-TVV		603255	DCC-TVV		1193713			
DEC		597428	DEC		1185975			
DEC-ARE		597432	DEC-ARE		1185982			
DEC-AVE		597586	DEC-AVE		1185926			
DEC-TVR		597379	DEC-TVR		1185928			
DEC-TVV		597379	DEC-TVV		1185928			

Table B.1: Full sample log-likelihood of entire period spans 3 January 1996 to 31 December 2012.

US Equities: Full sample ranking criterions

Model	k	AIC	BIC	Model	k	AIC	BIC	Model	k	AIC	BIC
$N = 5$				$N = 10$				$N = 25$			
CCC	15	-99200	-99105	CCC	30	-218098	-217907	CCC	75	-591797	-591320
DCC	17	-113828	-113720	DCC	32	-217662	-217458	DCC	77	-582710	-582220
DCC-ARE	18	-113833	-113719	DCC-ARE	33	-217771	-217561	DCC-ARE	78	-582426	-581930
DCC-AVE	18	-113832	-113717	DCC-AVE	33	-217735	-217525	DCC-AVE	78	-582679	-582183
DCC-TVR	20	-113828	-113700	DCC-TVR	35	-217632	-217410	DCC-TVR	80	-579972	-579463
DCC-TVV	20	-113828	-113700	DCC-TVV	35	-217614	-217392	DCC-TVV	80	-579973	-579465
DEC	17	-113804	-113696	DEC	32	-216565	-216362	DEC	77	-577253	-576764
DEC-ARE	18	-113803	-113688	DEC-ARE	33	-216566	-216356	DEC-ARE	78	-577259	-576763
DEC-AVE	18	-113811	-113696	DEC-AVE	33	-216566	-216357	DEC-AVE	78	-577362	-576866
DEC-TVR	20	-113765	-113637	DEC-TVR	35	-216562	-216339	DEC-TVR	80	-577192	-576684
DEC-TVV	20	-113765	-113637	DEC-TVV	35	-216562	-216339	DEC-TVV	80	-577192	-576684
$N = 50$				$N = 100$							
CCC	150	-1173924	-1172970	CCC	300	-2345091	-2343183				
DCC	152	-1219521	-1218555	DCC	302	-2435941	-2434021				
DCC-ARE	153	-1219780	-1218807	DCC-ARE	303	-2436015	-2434088				
DCC-AVE	153	-1219665	-1218692	DCC-AVE	303	-2436586	-2434659				
DCC-TVR	155	-1206194	-1205208	DCC-TVR	305	-2386818	-2384879				
DCC-TVV	155	-1206200	-1205214	DCC-TVV	305	-2386815	-2384876				
DEC	152	-1194551	-1193585	DEC	302	-2371347	-2369426				
DEC-ARE	153	-1194559	-1193586	DEC-ARE	303	-2371357	-2369430				
DEC-AVE	153	-1194865	-1193892	DEC-AVE	303	-2371245	-2369318				
DEC-TVR	155	-1194449	-1193463	DEC-TVR	305	-2371246	-2369306				
DEC-TVV	155	-1194449	-1193463	DEC-TVV	305	-2371246	-2369306				

Table B.2: AIC and BIC ranking criterion, US equities. Number of observations is 4268, k denotes the number of parameters.

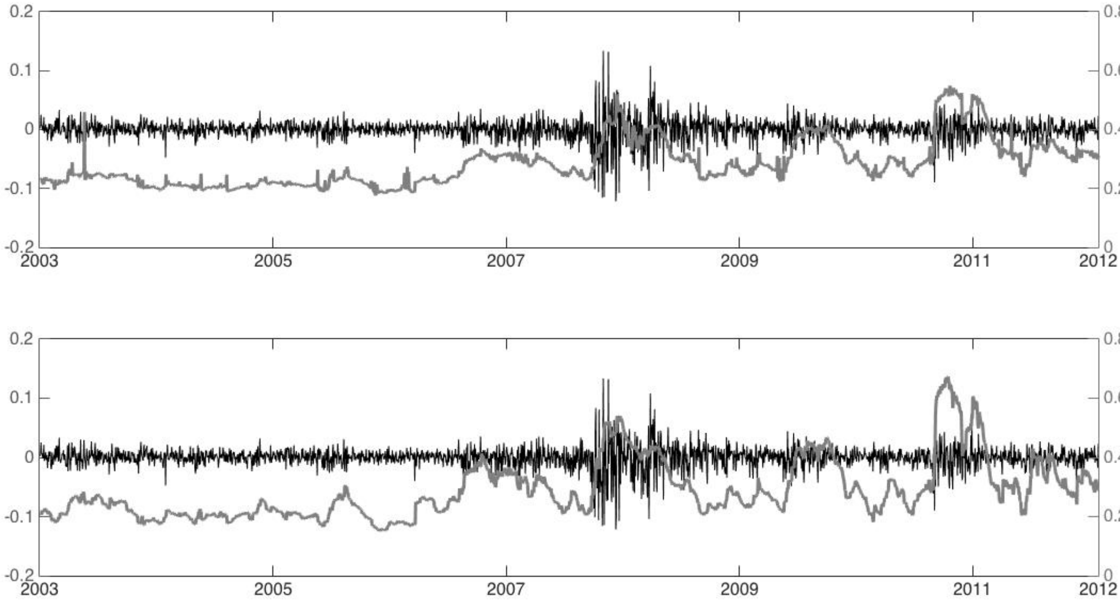

Figure B.1: Average daily return of portfolio of 10 US equities for out-of-sample period (left axis). One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC model (top, right axis). One-step-ahead equicorrelation forecasts, ρ_t (bottom, right axis). Entire period spans 3 January 1996 to 31 December 2012.

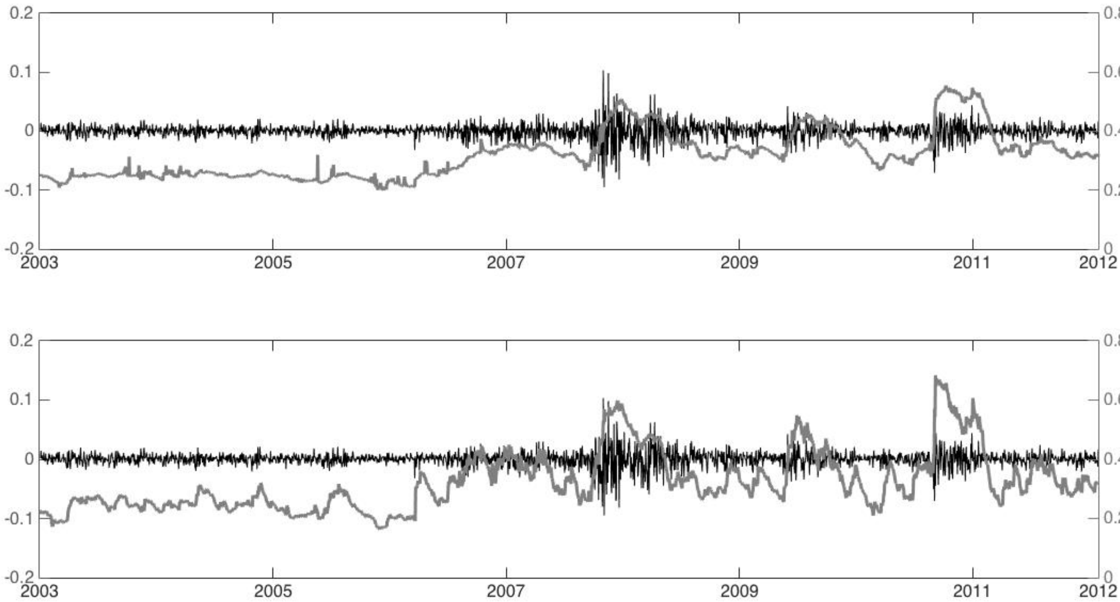

Figure B.2: Average daily return of portfolio of 50 US equities for out-of-sample period (left axis). One-step-ahead average forecasts of correlation, $\bar{\rho}_t$, for the cDCC model (top, right axis). One-step-ahead equicorrelation forecasts, ρ_t (bottom, right axis). Entire period spans 3 January 1996 to 31 December 2012.

US Equities: Average correlation forecasts, summary statistics

Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.
DCC	5	0.2651	0.0872	DCC	10	0.2839	0.0782	DCC	25	0.2939	0.0766
DCC-ARE		0.2661	0.0868	DCC-ARE		0.2814	0.0780	DCC-ARE		0.2926	0.0757
DCC-AVE		0.3147	0.0899	DCC-AVE		0.3125	0.0726	DCC-AVE		0.3314	0.0659
DCC-TVR		0.2645	0.0885	DCC-TVR		0.2839	0.0789	DCC-TVR		0.2932	0.0767
DCC-TVV		0.2668	0.0869	DCC-TVV		0.2842	0.0791	DCC-TVV		0.2931	0.0767
DEC		0.2703	0.1140	DEC		0.2903	0.1003	DEC		0.2986	0.0984
DEC-ARE		0.2743	0.1142	DEC-ARE		0.2941	0.1008	DEC-ARE		0.3108	0.1062
DEC-AVE		0.2757	0.1182	DEC-AVE		0.2871	0.1004	DEC-AVE		0.3712	0.1114
DEC-TVR		0.2553	0.0988	DEC-TVR		0.2774	0.0914	DEC-TVR		0.2953	0.0970
DEC-TVV		0.2543	0.1024	DEC-TVV		0.2802	0.0958	DEC-TVV		0.2942	0.0970
DCC	50	0.3196	0.0793	DCC	100	0.3149	0.0765				
DCC-ARE		0.3171	0.0772	DCC-ARE		0.3102	0.0747				
DCC-AVE		0.3446	0.0598	DCC-AVE		0.3419	0.0595				
DCC-TVR		0.3181	0.0793	DCC-TVR		0.3131	0.0752				
DCC-TVV		0.3186	0.0800	DCC-TVV		0.3144	0.0764				
DEC		0.3239	0.0995	DEC		0.3182	0.0944				
DEC-ARE		0.3452	0.1072	DEC-ARE		0.3347	0.0996				
DEC-AVE		0.3712	0.1143	DEC-AVE		0.3489	0.1120				
DEC-TVR		0.3215	0.1014	DEC-TVR		0.3200	0.0907				
DEC-TVV		0.3227	0.1037	DEC-TVV		0.3170	0.0974				

Table B.3: Out-of-sample mean, $\bar{\rho}$, and standard deviation for each equicorrelation model and average correlations of cDCC models. In-sample period of 2000 observations (Jan 1996 to Dec 2003), entire period spans 3 January 1996 to 31 December 2012.

US Equities: Average correlation forecasts, summary statistics, first low volatility sub-period

Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.
DCC	5	0.1992	0.0170	DCC	10	0.2176	0.0208	DCC	25	0.2270	0.0149
DCC-ARE		0.1997	0.0229	DCC-ARE		0.2154	0.0167	DCC-ARE		0.2256	0.0133
DCC-AVE		0.2885	0.0829	DCC-AVE		0.2788	0.0586	DCC-AVE		0.2908	0.0450
DCC-TVR		0.1974	0.0221	DCC-TVR		0.2164	0.0184	DCC-TVR		0.2258	0.0136
DCC-TVV		0.2024	0.0198	DCC-TVV		0.2164	0.0185	DCC-TVV		0.2258	0.0136
DEC		0.1899	0.0296	DEC		0.2115	0.0300	DEC		0.2187	0.0269
DEC-ARE		0.1969	0.0327	DEC-ARE		0.2165	0.0297	DEC-ARE		0.2192	0.0201
DEC-AVE		0.1961	0.0282	DEC-AVE		0.2104	0.0303	DEC-AVE		0.3021	0.0459
DEC-TVR		0.1957	0.0266	DEC-TVR		0.2143	0.0342	DEC-TVR		0.2243	0.0411
DEC-TVV		0.1961	0.0148	DEC-TVV		0.2142	0.0330	DEC-TVV		0.2233	0.0416
DCC	50	0.2448	0.0146	DCC	100	0.2415	0.0135				
DCC-ARE		0.2453	0.0112	DCC-ARE		0.2391	0.0125				
DCC-AVE		0.3018	0.0334	DCC-AVE		0.2983	0.0286				
DCC-TVR		0.2429	0.0134	DCC-TVR		0.2408	0.0124				
DCC-TVV		0.2427	0.0131	DCC-TVV		0.2409	0.0123				
DEC		0.2384	0.0315	DEC		0.2366	0.0279				
DEC-ARE		0.2600	0.0578	DEC-ARE		0.2512	0.0359				
DEC-AVE		0.2977	0.0681	DEC-AVE		0.2652	0.0573				
DEC-TVR		0.2407	0.0409	DEC-TVR		0.2454	0.0202				
DEC-TVV		0.2423	0.0495	DEC-TVV		0.2399	0.0457				

Table B.4: Out-of-sample mean, $\bar{\rho}$, and standard deviation for each equicorrelation model and average correlations of cDCC models. First low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Average correlation forecasts, summary statistics, high volatility sub-period

Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.
DCC	5	0.3014	0.0949	DCC	10	0.3182	0.0789	DCC	25	0.3296	0.0769
DCC-ARE		0.3028	0.0928	DCC-ARE		0.3154	0.0795	DCC-ARE		0.3284	0.0754
DCC-AVE		0.3333	0.0957	DCC-AVE		0.3312	0.0766	DCC-AVE		0.3576	0.0684
DCC-TVR		0.3015	0.0955	DCC-TVR		0.3189	0.0797	DCC-TVR		0.3291	0.0767
DCC-TVV		0.3023	0.0950	DCC-TVV		0.3195	0.0797	DCC-TVV		0.3291	0.0767
DEC		0.3182	0.1262	DEC		0.3358	0.1058	DEC		0.3464	0.1021
DEC-ARE		0.3207	0.1276	DEC-ARE		0.3397	0.1074	DEC-ARE		0.3690	0.1061
DEC-AVE		0.3191	0.1340	DEC-AVE		0.3334	0.1069	DEC-AVE		0.4155	0.1251
DEC-TVR		0.3015	0.1147	DEC-TVR		0.3113	0.1002	DEC-TVR		0.3379	0.1029
DEC-TVV		0.3023	0.1226	DEC-TVV		0.3151	0.1064	DEC-TVV		0.3366	0.1030
DCC	50	0.3637	0.0739	DCC	100	0.3591	0.0703				
DCC-ARE		0.3592	0.0737	DCC-ARE		0.3527	0.0695				
DCC-AVE		0.3722	0.0608	DCC-AVE		0.3726	0.0601				
DCC-TVR		0.3621	0.0738	DCC-TVR		0.3563	0.0690				
DCC-TVV		0.3632	0.0745	DCC-TVV		0.3587	0.0701				
DEC		0.3800	0.0973	DEC		0.3722	0.0918				
DEC-ARE		0.4056	0.1015	DEC-ARE		0.3934	0.0959				
DEC-AVE		0.4187	0.1214	DEC-AVE		0.4088	0.1121				
DEC-TVR		0.3744	0.1028	DEC-TVR		0.3681	0.0918				
DEC-TVV		0.3763	0.1043	DEC-TVV		0.3682	0.0969				

Table B.5: Out-of-sample mean, $\bar{\rho}$, and standard deviation for each equicorrelation model and average correlations of cDCC models. High volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Average correlation forecasts, summary statistics, second low volatility sub-period

Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.	Model	N	$\bar{\rho}$	s.d.
DCC	5	0.3018	0.0522	DCC	10	0.3315	0.0415	DCC	25	0.3361	0.0387
DCC-ARE		0.3023	0.0528	DCC-ARE		0.3299	0.0404	DCC-ARE		0.3348	0.0377
DCC-AVE		0.3086	0.0533	DCC-AVE		0.3310	0.0512	DCC-AVE		0.3357	0.0385
DCC-TVR		0.3018	0.0523	DCC-TVR		0.3316	0.0415	DCC-TVR		0.3361	0.0386
DCC-TVV		0.3018	0.0523	DCC-TVV		0.3316	0.0415	DCC-TVV		0.3361	0.0386
DEC		0.2979	0.0720	DEC		0.3236	0.0549	DEC		0.3239	0.0517
DEC-ARE		0.2989	0.0737	DEC-ARE		0.3237	0.0549	DEC-ARE		0.3239	0.0516
DEC-AVE		0.3220	0.0741	DEC-AVE		0.3097	0.0565	DEC-AVE		0.3787	0.0709
DEC-TVR		0.2760	0.0669	DEC-TVR		0.3165	0.0581	DEC-TVR		0.3178	0.0550
DEC-TVV		0.2754	0.0660	DEC-TVV		0.3238	0.0550	DEC-TVV		0.3170	0.0540
DCC	50	0.3470	0.0394	DCC	100	0.3371	0.0350				
DCC-ARE		0.3438	0.0373	DCC-ARE		0.3337	0.0327				
DCC-AVE		0.3490	0.0367	DCC-AVE		0.3332	0.0348				
DCC-TVR		0.3469	0.0392	DCC-TVR		0.3370	0.0348				
DCC-TVV		0.3469	0.0392	DCC-TVV		0.3370	0.0348				
DEC		0.3271	0.0437	DEC		0.3197	0.0427				
DEC-ARE		0.3271	0.0437	DEC-ARE		0.3197	0.0427				
DEC-AVE		0.3781	0.0602	DEC-AVE		0.3283	0.0436				
DEC-TVR		0.3249	0.0442	DEC-TVR		0.3271	0.0566				
DEC-TVV		0.3216	0.0454	DEC-TVV		0.3169	0.0450				

Table B.6: Out-of-sample mean, $\bar{\rho}$, and standard deviation for each equicorrelation model and average correlations of cDCC models. Second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4268).

US Equities: Relative economic value, entire period, N = 5

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	12.824 0.636	63.131 0.928	17.144 0.694	13.806 0.650	14.714 0.650	25.508 0.818	36.768 0.876	51.648 0.914	22.773 0.798	30.599 0.852
DCC		-	50.950 0.942	4.446 0.896	0.689 0.572	2.030 0.626	14.418 0.828	25.265 0.880	38.140 0.912	11.718 0.786	16.506 0.816
DCC-AVE			-	−51.431 0.052	−55.391 0.050	−53.941 0.050	−40.549 0.050	−29.424 0.088	−16.588 0.248	−43.181 0.042	−38.525 0.076
DCC-TVV				-	−3.842 0.230	−2.437 0.146	9.949 0.788	20.817 0.856	33.682 0.894	7.251 0.748	11.986 0.780
DCC-ARE					-	1.094 0.524	13.303 0.792	24.088 0.864	36.947 0.904	10.617 0.768	15.464 0.790
DCC-TVR						-	12.341 0.808	23.198 0.864	36.064 0.898	9.634 0.772	14.366 0.798
DECO							-	10.676 0.932	22.722 0.960	−2.749 0.124	0.662 0.600
DEC-AVE								-	11.982 0.916	−14.093 0.048	−10.295 0.114
DEC-TVV									-	−27.753 0.028	−22.389 0.002
DEC-ARE										-	3.298 0.674
DEC-TVR											-

Table B.7: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 5 assets.

US Equities: Relative economic value, entire period, N = 10

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−6.108 0.400	−7.818 0.456	−5.846 0.422	−1.840 0.470	−9.370 0.376	26.159 0.850	26.161 0.832	33.199 0.850	24.790 0.830	37.345 0.862
DCC		-	−1.912 0.486	0.181 0.520	4.062 0.802	−3.554 0.210	30.088 0.830	29.395 0.806	36.630 0.846	28.705 0.818	40.399 0.848
DCC-AVE			-	−0.112 0.506	3.777 0.566	−3.846 0.418	30.464 0.856	29.950 0.830	37.226 0.870	29.083 0.844	40.982 0.884
DCC-TVV				-	3.877 0.906	−3.735 0.034	29.863 0.824	29.160 0.798	36.360 0.848	28.489 0.808	40.131 0.852
DCC-ARE					-	−7.648 0.036	25.932 0.800	25.253 0.778	32.452 0.834	24.559 0.780	36.233 0.844
DCC-TVR						-	33.649 0.850	32.942 0.818	40.137 0.858	32.273 0.834	43.913 0.868
DECO							-	−1.014 0.512	6.230 0.754	−1.410 0.344	9.870 0.816
DEC-AVE								-	6.706 0.838	−1.341 0.384	10.525 0.912
DEC-TVV									-	−8.553 0.220	3.551 0.854
DEC-ARE										-	11.124 0.832
DEC-TVR											-

Table B.8: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 10 assets.

US Equities: Relative economic value, entire period, N = 25

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−39.659 0.058	−24.820 0.056	−41.177 0.046	−36.429 0.076	−40.201 0.060	−7.871 0.360	−3.834 0.452	−0.551 0.510	−3.795 0.442	−1.339 0.486
DCC		-	14.099 0.806	−1.552 0.214	3.174 0.858	−0.594 0.400	29.921 0.820	34.316 0.862	37.133 0.890	34.061 0.856	36.340 0.890
DCC-AVE			-	−17.178 0.134	−12.437 0.218	−16.203 0.148	15.190 0.724	19.870 0.776	22.463 0.808	19.401 0.772	21.678 0.802
DCC-TVV				-	4.726 0.996	0.958 0.952	31.453 0.832	35.835 0.868	38.665 0.896	35.595 0.864	37.874 0.890
DCC-ARE					-	−3.787 0.016	26.733 0.800	31.116 0.838	33.948 0.860	30.881 0.822	33.157 0.848
DCC-TVR						-	30.501 0.824	34.884 0.858	37.712 0.890	34.642 0.852	36.921 0.888
DECO							-	4.564 0.646	7.165 0.994	4.171 0.942	6.381 0.986
DEC-AVE								-	2.197 0.616	−0.713 0.502	1.421 0.584
DEC-TVV									-	−3.147 0.184	−0.790 0.076
DEC-ARE										-	2.116 0.708
DEC-TVR											-

Table B.9: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 25 assets.

US Equities: Relative economic value, entire period, N = 100

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−2.303 0.466	−22.215 0.170	−10.281 0.342	−11.957 0.320	−4.079 0.432	17.426 0.736	19.418 0.756	18.310 0.754	15.778 0.716	12.953 0.680
DCC		-	−20.748 0.020	−8.001 0.002	−9.737 0.000	−1.882 0.264	17.426 0.672	19.452 0.696	18.314 0.684	15.774 0.656	12.933 0.628
DCC-AVE			-	11.727 0.864	10.027 0.848	17.889 0.960	38.258 0.852	40.311 0.876	39.142 0.862	36.620 0.842	33.769 0.824
DCC-TVV				-	−1.739 0.264	6.119 1.000	25.389 0.756	27.413 0.766	26.276 0.768	23.737 0.732	20.898 0.708
DCC-ARE					-	7.820 0.986	27.130 0.762	29.152 0.768	28.017 0.764	25.478 0.748	22.640 0.732
DCC-TVR						-	19.338 0.698	21.360 0.716	20.224 0.714	17.686 0.684	14.848 0.658
DECO							-	2.080 0.836	0.873 0.868	−1.655 0.110	−4.531 0.000
DEC-AVE								-	−1.235 0.300	−3.763 0.004	−6.647 0.000
DEC-TVV									-	−2.539 0.038	−5.415 0.000
DEC-ARE										-	−2.892 0.070
DEC-TVR											-

Table B.10: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 100 assets.

US Equities: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	13.120 (0.946)	−86.309 (0.038)	13.444 (0.918)	2.786 (0.598)	3.165 (0.600)	−4.537 (0.336)	−7.581 (0.266)	−9.838 (0.172)	−8.858 (0.262)	−10.401 (0.194)
DCC		-	−99.380 (0.018)	0.338 (0.548)	−10.204 (0.092)	−9.899 (0.064)	−17.919 (0.170)	−20.994 (0.092)	−23.282 (0.066)	−22.202 (0.116)	−23.817 (0.084)
DCC-AVE			-	93.663 (0.990)	82.965 (0.960)	83.207 (0.964)	75.051 (0.916)	72.232 (0.932)	69.900 (0.930)	70.964 (0.924)	69.410 (0.932)
DCC-TVV				-	−10.555 (0.134)	−10.263 (0.096)	−18.312 (0.166)	−21.407 (0.090)	−23.701 (0.076)	−22.566 (0.112)	−24.228 (0.088)
DCC-ARE					-	0.219 (0.434)	−7.991 (0.308)	−11.120 (0.268)	−13.432 (0.214)	−12.274 (0.278)	−13.949 (0.222)
DCC-TVR						-	−8.160 (0.292)	−11.281 (0.230)	−13.585 (0.176)	−12.438 (0.256)	−14.110 (0.192)
DECO							-	−3.101 (0.404)	−5.482 (0.336)	−4.325 (0.340)	−5.942 (0.182)
DEC-AVE								-	−2.431 (0.272)	−1.395 (0.402)	−2.865 (0.130)
DEC-TVV									-	0.984 (0.474)	−0.517 (0.370)
DEC-ARE										-	−1.693 (0.380)
DEC-TVR											-

Table B.11: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 5 assets, first low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−29.061 (0.192)	−95.934 (0.074)	−30.122 (0.202)	−19.922 (0.246)	−30.158 (0.200)	21.334 (0.698)	19.908 (0.682)	22.721 (0.722)	12.984 (0.624)	22.899 (0.724)
DCC		-	−67.243 (0.078)	−1.106 (0.474)	8.959 (0.870)	−1.142 (0.472)	49.640 (0.772)	48.182 (0.766)	51.004 (0.792)	41.306 (0.726)	51.187 (0.796)
DCC-AVE			-	63.545 (0.866)	73.658 (0.920)	63.510 (0.866)	114.689 (0.912)	113.225 (0.916)	116.164 (0.924)	106.400 (0.908)	116.351 (0.926)
DCC-TVV				-	10.062 (0.878)	−0.036 (0.140)	50.755 (0.772)	49.298 (0.770)	52.107 (0.798)	42.422 (0.734)	52.291 (0.798)
DCC-ARE					-	−10.159 (0.118)	40.686 (0.726)	39.232 (0.726)	42.042 (0.758)	32.359 (0.688)	42.224 (0.758)
DCC-TVR						-	50.790 (0.772)	49.334 (0.770)	52.143 (0.798)	42.458 (0.734)	52.326 (0.798)
DECO							-	−1.468 (0.460)	1.413 (0.594)	−8.401 (0.028)	1.602 (0.602)
DEC-AVE								-	2.780 (0.602)	−7.061 (0.254)	2.967 (0.606)
DEC-TVV									-	−9.949 (0.160)	0.191 (0.740)
DEC-ARE										-	9.915 (0.838)
DEC-TVR											-

Table B.12: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 10 assets, first low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−51.714 0.056	−75.360 0.016	−49.078 0.064	−48.782 0.056	−49.016 0.064	−16.674 0.302	−50.034 0.064	−21.925 0.224	−19.294 0.266	−22.372 0.212
DCC		-	−23.953 0.048	2.627 0.816	2.904 0.812	2.689 0.824	34.615 0.798	1.323 0.530	29.370 0.778	31.983 0.786	28.925 0.774
DCC-AVE			-	25.574 0.970	25.848 0.964	25.636 0.972	57.721 0.900	24.835 0.770	52.519 0.888	55.083 0.894	52.081 0.888
DCC-TVV				-	0.277 0.562	0.063 0.992	31.970 0.778	−1.330 0.498	26.725 0.758	29.340 0.766	26.280 0.760
DCC-ARE					-	−0.225 0.450	31.693 0.780	−1.616 0.500	26.445 0.760	29.062 0.770	26.000 0.760
DCC-TVR						-	31.909 0.778	−1.392 0.498	26.664 0.758	29.277 0.766	26.218 0.760
DECO							-	−33.345 0.040	−5.214 0.046	−2.656 0.078	−5.661 0.046
DEC-AVE								-	27.699 0.940	30.195 0.940	27.259 0.934
DEC-TVV									-	2.490 0.754	−0.448 0.250
DEC-ARE										-	−3.043 0.226
DEC-TVR											-

Table B.13: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 25 assets, first low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−40.979 0.120	−48.309 0.104	−33.799 0.150	−25.569 0.196	−32.722 0.152	−12.748 0.418	−15.728 0.420	−11.867 0.438	−9.405 0.468	−12.962 0.424
DCC		-	−7.845 0.300	7.120 0.836	15.177 0.972	8.191 0.840	27.460 0.678	24.517 0.658	28.346 0.682	30.815 0.704	27.249 0.678
DCC-AVE			-	14.032 0.782	22.216 0.910	15.113 0.792	34.951 0.724	32.129 0.722	35.848 0.724	38.384 0.740	34.747 0.724
DCC-TVV				-	8.058 0.936	1.071 0.688	20.278 0.648	17.329 0.638	21.163 0.648	23.628 0.668	20.066 0.642
DCC-ARE					-	−7.068 0.088	12.302 0.596	9.354 0.590	13.184 0.600	15.653 0.636	12.087 0.598
DCC-TVR						-	19.207 0.642	16.258 0.636	20.091 0.642	22.557 0.664	18.994 0.630
DECO							-	−2.897 0.360	0.919 0.664	3.423 0.680	−0.194 0.454
DEC-AVE								-	3.642 0.682	6.190 0.756	2.527 0.634
DEC-TVV									-	2.478 0.648	−1.124 0.218
DEC-ARE										-	−3.682 0.302
DEC-TVR											-

Table B.14: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 50 assets, first low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	6.724 0.554	−32.753 0.140	−5.143 0.398	−11.023 0.308	−3.740 0.416	5.633 0.550	−2.152 0.490	5.927 0.552	4.244 0.538	2.946 0.530
DCC		-	−39.990 0.000	−11.880 0.004	−17.859 0.012	−10.476 0.016	−1.831 0.506	−9.605 0.442	−1.538 0.512	−3.218 0.498	−4.517 0.482
DCC-AVE			-	27.411 0.976	21.506 0.952	28.814 0.986	37.982 0.780	30.232 0.732	38.278 0.776	36.610 0.776	35.305 0.770
DCC-TVV				-	−5.983 0.186	1.402 0.994	10.026 0.594	2.251 0.532	10.318 0.598	8.637 0.586	7.341 0.574
DCC-ARE					-	7.325 0.852	16.047 0.642	8.272 0.588	16.340 0.646	14.660 0.628	13.362 0.622
DCC-TVR						-	8.623 0.584	0.848 0.528	8.915 0.588	7.234 0.576	5.937 0.566
DECO							-	−7.767 0.050	0.303 0.610	−1.379 0.232	−2.693 0.030
DEC-AVE								-	8.037 0.948	6.357 0.932	5.041 0.908
DEC-TVV									-	−1.691 0.158	−3.008 0.060
DEC-ARE										-	−1.322 0.148
DEC-TVR											-

Table B.15: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 100 assets, first low volatility sub-period of Dec 2003 to Feb 2007 (2001:2806).

US Equities: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	144.755 (0.986)	115.475 (0.938)	146.129 (0.984)	150.499 (0.982)	145.832 (0.986)	221.305 (0.988)	249.469 (0.992)	240.018 (0.982)	213.924 (0.984)	228.559 (0.988)
DCC		-	−29.212 (0.206)	1.438 (0.662)	5.804 (0.962)	1.138 (0.536)	79.525 (0.900)	108.588 (0.936)	96.009 (0.896)	72.415 (0.894)	85.320 (0.900)
DCC-AVE			-	28.787 (0.784)	33.244 (0.830)	28.454 (0.762)	107.044 (0.904)	136.143 (0.934)	123.435 (0.904)	99.959 (0.900)	112.808 (0.912)
DCC-TVV				-	4.337 (0.898)	−0.316 (0.412)	78.020 (0.894)	107.092 (0.926)	94.494 (0.896)	70.913 (0.888)	83.803 (0.894)
DCC-ARE					-	−4.711 (0.192)	73.702 (0.894)	102.765 (0.926)	90.160 (0.892)	66.603 (0.882)	79.495 (0.890)
DCC-TVR						-	78.327 (0.898)	107.395 (0.930)	94.811 (0.896)	71.221 (0.890)	84.118 (0.896)
DECO							-	29.440 (0.996)	15.619 (0.784)	−7.029 (0.070)	5.211 (0.626)
DEC-AVE								-	−14.291 (0.272)	−36.913 (0.008)	−24.735 (0.092)
DEC-TVV									-	−24.282 (0.176)	−10.962 (0.222)
DEC-ARE										-	12.002 (0.724)
DEC-TVR											-

Table B.16: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 5 assets, high volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	104.306 (0.860)	79.622 (0.820)	108.355 (0.874)	104.852 (0.862)	111.516 (0.876)	84.304 (0.948)	84.061 (0.938)	96.582 (0.956)	86.026 (0.946)	82.884 (0.952)
DCC		-	−26.110 (0.062)	4.118 (0.852)	0.358 (0.482)	7.235 (0.936)	−29.202 (0.380)	−30.203 (0.382)	−17.343 (0.446)	−27.366 (0.384)	−31.630 (0.380)
DCC-AVE			-	28.651 (0.966)	24.897 (0.938)	31.769 (0.966)	−3.413 (0.504)	−4.174 (0.488)	8.492 (0.570)	−1.588 (0.510)	−5.678 (0.484)
DCC-TVV				-	−3.762 (0.188)	3.117 (0.864)	−33.367 (0.366)	−34.390 (0.366)	−21.529 (0.412)	−31.529 (0.366)	−35.822 (0.360)
DCC-ARE					-	6.852 (0.884)	−29.620 (0.374)	−30.622 (0.384)	−17.770 (0.450)	−27.779 (0.382)	−32.051 (0.378)
DCC-TVR						-	−36.473 (0.344)	−37.493 (0.350)	−24.632 (0.396)	−34.637 (0.356)	−38.921 (0.342)
DECO							-	−0.728 (0.502)	11.640 (0.748)	1.884 (0.618)	−2.483 (0.472)
DEC-AVE								-	12.031 (0.774)	2.244 (0.564)	−1.972 (0.458)
DEC-TVV									-	−10.244 (0.318)	−14.164 (0.036)
DEC-ARE										-	−4.469 (0.452)
DEC-TVR											-

Table B.17: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 10 assets, high volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	27.953 0.780	42.615 0.806	27.330 0.774	23.540 0.730	27.285 0.774	28.979 0.712	25.459 0.656	23.856 0.682	26.704 0.690	22.762 0.666
DCC		-	14.346 0.784	−0.625 0.260	−4.427 0.030	−0.674 0.260	−2.775 0.508	−5.841 0.478	−8.053 0.472	−4.963 0.488	−9.105 0.468
DCC-AVE			-	−15.679 0.194	−19.477 0.162	−15.727 0.194	−17.398 0.444	−20.367 0.406	−22.661 0.396	−19.533 0.414	−23.708 0.384
DCC-TVV				-	−3.802 0.030	−0.049 0.446	−2.155 0.506	−5.223 0.484	−7.433 0.474	−4.344 0.504	−8.486 0.470
DCC-ARE					-	3.741 0.954	1.649 0.532	−1.423 0.502	−3.630 0.494	−0.536 0.516	−4.682 0.490
DCC-TVR						-	−2.102 0.510	−5.171 0.482	−7.381 0.476	−4.290 0.508	−8.433 0.470
DECO							-	−3.109 0.382	−5.222 0.152	−2.152 0.360	−6.295 0.088
DEC-AVE								-	−2.442 0.420	0.767 0.546	−3.503 0.366
DEC-TVV									-	2.995 0.704	−1.080 0.132
DEC-ARE										-	−4.274 0.236
DEC-TVR											-

Table B.18: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 25 assets, high volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−13.333 0.326	−1.486 0.478	−15.083 0.312	−12.995 0.334	−16.595 0.298	68.960 0.870	71.456 0.860	65.560 0.860	70.013 0.862	69.458 0.872
DCC		-	10.673 0.778	−1.767 0.266	0.296 0.518	−3.296 0.212	79.169 0.890	81.853 0.884	75.699 0.872	80.271 0.882	79.638 0.892
DCC-AVE			-	−13.496 0.168	−11.418 0.204	−15.016 0.138	68.581 0.860	71.241 0.862	65.138 0.854	69.700 0.856	69.060 0.862
DCC-TVV				-	2.062 0.798	−1.529 0.224	80.931 0.890	83.615 0.894	77.461 0.882	82.033 0.886	81.400 0.894
DCC-ARE					-	−3.613 0.130	78.866 0.888	81.544 0.884	75.396 0.872	79.967 0.876	79.335 0.892
DCC-TVR						-	82.462 0.900	85.149 0.894	78.993 0.890	83.561 0.886	82.932 0.900
DECO							-	2.691 0.706	−3.442 0.052	1.155 0.596	0.473 0.572
DEC-AVE								-	−6.244 0.088	−1.620 0.322	−2.326 0.336
DEC-TVV									-	4.575 0.840	3.906 1.000
DEC-ARE										-	−0.738 0.456
DEC-TVR											-

Table B.19: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 50 assets, high volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−40.853 0.078	−46.372 0.024	−44.378 0.066	−36.332 0.098	−45.555 0.060	57.434 0.906	54.142 0.892	55.975 0.902	56.509 0.896	57.877 0.914
DCC		-	−6.071 0.344	−3.532 0.056	4.479 0.940	−4.753 0.088	96.288 0.986	93.011 0.982	94.812 0.984	95.384 0.984	96.744 0.984
DCC-AVE			-	1.911 0.580	9.931 0.752	0.715 0.540	102.279 0.998	99.014 0.994	100.805 0.996	101.376 0.994	102.728 0.998
DCC-TVV				-	8.008 1.000	−1.221 0.294	99.817 0.990	96.539 0.988	98.341 0.990	98.913 0.988	100.273 0.990
DCC-ARE					-	−9.252 0.006	91.809 0.984	88.529 0.982	90.333 0.984	90.904 0.982	92.265 0.984
DCC-TVR						-	101.054 0.992	97.776 0.990	99.578 0.992	100.150 0.992	101.510 0.992
DECO							-	−3.262 0.008	−1.466 0.020	−0.905 0.326	0.451 0.692
DEC-AVE								-	1.781 0.868	2.348 0.970	3.695 0.952
DEC-TVV									-	0.557 0.568	1.913 0.918
DEC-ARE										-	1.340 0.716
DEC-TVR											-

Table B.20: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 100 assets, high volatility sub-period of Mar 2007 to Dec 2011 (2807:4019).

US Equities: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−114.855 0.046	−124.459 0.042	−115.037 0.046	−115.898 0.046	−115.073 0.046	−135.886 0.040	−143.656 0.086	−108.157 0.000	−136.824 0.040	−109.346 0.000
DCC		-	−9.535 0.134	−0.182 0.372	−1.036 0.170	−0.218 0.376	−22.037 0.360	−29.603 0.392	5.301 0.522	−22.958 0.358	4.125 0.518
DCC-AVE			-	9.342 0.870	8.489 0.846	9.306 0.872	−12.514 0.428	−20.040 0.426	14.784 0.576	−13.433 0.418	13.609 0.574
DCC-TVV				-	−0.855 0.196	−0.036 0.328	−21.856 0.362	−29.422 0.392	5.482 0.524	−22.776 0.360	4.307 0.524
DCC-ARE					-	0.818 0.814	−21.001 0.368	−28.564 0.398	6.330 0.524	−21.921 0.362	5.154 0.524
DCC-TVR						-	−21.820 0.362	−29.385 0.392	5.517 0.524	−22.740 0.360	4.341 0.524
DECO							-	−7.545 0.438	27.292 0.700	−0.915 0.366	26.123 0.696
DEC-AVE								-	34.407 0.638	6.332 0.548	33.239 0.634
DEC-TVV									-	−29.013 0.292	−1.175 0.076
DEC-ARE										-	27.013 0.694
DEC-TVR											-

Table B.21: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 5 assets, second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4271).

US Equities: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−45.094 0.256	37.908 0.700	−45.318 0.256	−41.390 0.272	−45.328 0.256	−183.497 0.006	−170.691 0.004	−183.240 0.006	−183.273 0.006	−179.173 0.004
DCC		-	82.578 0.980	−0.223 0.056	3.702 0.962	−0.233 0.048	−140.570 0.074	−127.926 0.092	−140.313 0.074	−140.346 0.074	−136.353 0.076
DCC-AVE			-	−83.842 0.018	−79.917 0.020	−83.852 0.018	−223.661 0.008	−210.933 0.012	−223.403 0.008	−223.437 0.008	−219.364 0.008
DCC-TVV				-	3.925 0.966	−0.010 0.060	−140.349 0.076	−127.705 0.092	−140.092 0.076	−140.125 0.076	−136.131 0.076
DCC-ARE					-	−3.937 0.032	−144.299 0.072	−131.643 0.086	−144.041 0.072	−144.074 0.072	−140.073 0.072
DCC-TVR						-	−140.339 0.076	−127.694 0.092	−140.082 0.076	−140.114 0.076	−136.121 0.076
DECO							-	12.624 0.740	0.260 0.842	0.225 0.884	4.208 0.582
DEC-AVE								-	−12.535 0.264	−12.568 0.258	−8.441 0.002
DEC-TVV									-	−0.035 0.336	3.947 0.568
DEC-ARE										-	3.983 0.576
DEC-TVR											-

Table B.22: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 10 assets, second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4271).

US Equities: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−0.600 0.516	0.259 0.520	−0.488 0.518	0.814 0.526	−0.483 0.518	−103.476 0.028	−128.071 0.006	−102.928 0.034	−103.571 0.028	−103.919 0.032
DCC		-	0.854 1.000	0.112 0.738	1.419 0.960	0.117 0.744	−104.620 0.092	−129.090 0.042	−104.107 0.098	−104.716 0.092	−105.097 0.096
DCC-AVE			-	−0.742 0.004	0.565 0.758	−0.738 0.004	−105.467 0.090	−129.938 0.042	−104.954 0.098	−105.563 0.090	−105.944 0.094
DCC-TVV				-	1.307 0.946	0.004 0.210	−104.731 0.092	−129.201 0.042	−104.219 0.098	−104.827 0.090	−105.209 0.096
DCC-ARE					-	−1.303 0.052	−106.048 0.090	−130.522 0.042	−105.535 0.096	−106.144 0.090	−106.525 0.090
DCC-TVR						-	−104.735 0.092	−129.206 0.042	−104.223 0.098	−104.831 0.090	−105.213 0.096
DECO							-	−24.501 0.036	0.530 0.562	−0.096 0.090	−0.456 0.474
DEC-AVE								-	24.906 0.920	24.301 0.964	23.921 0.918
DEC-TVV									-	−0.646 0.432	−0.986 0.002
DEC-ARE										-	−0.360 0.488
DEC-TVR											-

Table B.23: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 25 assets, second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4271).

US Equities: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	29.342 0.726	8.659 0.578	29.168 0.726	31.333 0.730	29.121 0.726	−61.955 0.172	−75.740 0.136	−60.388 0.176	−61.981 0.172	−61.715 0.172
DCC		-	−21.162 0.306	−0.175 0.196	1.992 0.922	−0.223 0.132	−93.066 0.170	−106.772 0.120	−91.511 0.170	−93.091 0.168	−92.831 0.168
DCC-AVE			-	20.315 0.684	22.479 0.710	20.268 0.684	−71.967 0.196	−85.657 0.144	−70.411 0.206	−71.992 0.196	−71.731 0.198
DCC-TVV				-	2.167 0.936	−0.048 0.152	−92.890 0.170	−106.597 0.120	−91.334 0.170	−92.916 0.170	−92.655 0.170
DCC-ARE					-	−2.216 0.060	−95.070 0.156	−108.779 0.116	−93.513 0.164	−95.095 0.154	−94.834 0.154
DCC-TVR						-	−92.842 0.170	−106.548 0.120	−91.286 0.170	−92.867 0.170	−92.607 0.170
DECO							-	−13.622 0.040	1.531 0.698	−0.026 0.192	0.228 0.580
DEC-AVE								-	15.099 0.932	13.550 0.956	13.801 0.946
DEC-TVV									-	−1.566 0.296	−1.309 0.240
DEC-ARE										-	0.254 0.584
DEC-TVR											-

Table B.24: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 50 assets, second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4271).

US Equities: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	33.851 0.792	34.367 0.802	33.780 0.792	35.431 0.802	33.772 0.792	−77.151 0.096	−78.011 0.094	−76.445 0.100	−77.168 0.096	−80.957 0.092
DCC		-	0.506 0.686	−0.071 0.274	1.587 0.950	−0.079 0.258	−112.136 0.078	−112.990 0.080	−111.435 0.080	−112.153 0.078	−115.941 0.074
DCC-AVE			-	−0.578 0.284	1.081 0.834	−0.586 0.282	−112.639 0.078	−113.494 0.076	−111.938 0.080	−112.656 0.078	−116.444 0.072
DCC-TVV				-	1.659 0.964	−0.008 0.088	−112.065 0.078	−112.919 0.078	−111.364 0.080	−112.081 0.078	−115.870 0.074
DCC-ARE					-	−1.667 0.036	−113.741 0.078	−114.596 0.074	−113.040 0.078	−113.758 0.078	−117.546 0.070
DCC-TVR						-	−112.057 0.078	−112.911 0.080	−111.356 0.080	−112.073 0.078	−115.863 0.074
DECO							-	−0.853 0.176	0.699 0.844	−0.016 0.012	−3.796 0.000
DEC-AVE								-	1.553 0.938	0.836 0.812	−2.944 0.000
DEC-TVV									-	−0.717 0.146	−4.498 0.000
DEC-ARE										-	−3.781 0.000
DEC-TVR											-

Table B.25: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of 100 assets, second low volatility sub-period of Dec 2011 to Dec 2012 (4020:4271).

European Indices: Additional Results

European Indices: Full Sample Ranking Criterions

Model	k	AIC	BIC
CCC	42	-122050	-121786
DCC	44	-353049	-352773
DCC-AVE	45	-352138	-351856
DCC-TVV	47	-352385	-352102
DCC-ARE	45	-353303	-353009
DCC-TVR	47	-353287	-352992
DEC	44	-340011	-339735
DEC-AVE	45	-340141	-339859
DEC-TVV	47	-340037	-339754
DEC-ARE	45	-339846	-339551
DEC-TVR	47	-339846	-339551

Table B.26: AIC and BIC ranking criterions, European indices. Number of observations is 4000.

European Indices: Relative economic value, all forecasts

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	17.951 0.860	30.076 0.962	5.997 0.698	13.408 0.806	−7409.905 0.788	−97.753 0.006	−71.476 0.012	−90.961 0.006	−103.375 0.000	−90.734 0.006
DCC		-	12.193 0.992	−11.884 0.000	−4.741 0.022	−7427.746 0.524	−117.452 0.002	−91.179 0.020	−110.623 0.004	−123.387 0.000	−110.402 0.004
DCC-AVE			-	−24.128 0.000	−17.006 0.010	−7440.022 0.242	−129.653 0.000	−103.367 0.006	−122.831 0.002	−135.604 0.000	−122.610 0.002
DCC-TVV				-	7.054 0.980	−7415.936 0.880	−105.670 0.006	−79.404 0.038	−98.843 0.012	−111.612 0.004	−98.621 0.012
DCC-ARE					-	−7423.041 0.674	−112.774 0.002	−86.504 0.024	−105.938 0.010	−118.682 0.002	−105.717 0.010
DCC-TVR						-	115.441 0.030	141.657 0.056	122.261 0.040	109.487 0.030	122.480 0.040
DECO							-	26.353 1.000	6.763 0.980	−6.339 0.006	6.978 0.990
DEC-AVE								-	−19.699 0.000	−32.835 0.000	−19.485 0.000
DEC-TVV									-	−13.198 0.006	0.213 0.846
DEC-ARE										-	13.236 0.994
DEC-TVR											-

Table B.27: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of European indices.

European Indices: Relative economic value, first low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−17.324 0.096	11.261 0.746	−98.670 0.000	−15.487 0.118	−19580.721 0.502	−239.677 0.000	−145.885 0.002	−205.392 0.000	−234.938 0.000	−205.084 0.000
DCC		-	28.830 0.992	−81.026 0.000	1.781 0.920	−19557.830 0.580	−222.715 0.000	−129.122 0.016	−188.398 0.000	−218.160 0.000	−188.103 0.002
DCC-AVE			-	−110.018 0.000	−27.239 0.008	−19587.374 0.426	−251.746 0.000	−158.152 0.004	−217.408 0.000	−247.220 0.000	−217.118 0.000
DCC-TVV				-	82.425 1.000	−19477.240 0.920	−142.151 0.004	−48.614 0.200	−107.838 0.012	−137.626 0.004	−107.543 0.014
DCC-ARE					-	−19559.946 0.570	−224.518 0.000	−130.916 0.016	−190.202 0.000	−219.953 0.000	−189.906 0.000
DCC-TVR						-	3503.966 0.046	3595.450 0.072	3538.174 0.052	3507.743 0.046	3538.398 0.052
DECO							-	93.169 1.000	34.268 0.998	4.265 0.916	34.536 0.998
DEC-AVE								-	−59.125 0.000	−89.195 0.000	−58.856 0.000
DEC-TVV									-	−30.172 0.006	0.260 0.738
DEC-ARE										-	30.175 0.996
DEC-TVR											-

Table B.28: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of European indices, first low volatility sub-period of Dec 2005 to Jul 2007 (2001:2359).

European Indices: Relative economic value, high volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	71.789 1.000	69.242 0.998	73.590 1.000	65.646 1.000	63.415 0.996	72.422 0.962	56.201 0.928	71.652 0.952	67.049 0.960	71.919 0.952
DCC		-	−2.354 0.390	1.778 0.670	−6.595 0.092	−8.378 0.016	−0.408 0.524	−16.651 0.310	−1.399 0.516	−6.128 0.400	−1.131 0.520
DCC-AVE			-	4.099 0.852	−4.321 0.274	−6.067 0.190	1.934 0.516	−14.295 0.326	0.929 0.518	−3.812 0.432	1.196 0.524
DCC-TVV				-	−8.418 0.116	−10.200 0.110	−2.234 0.460	−18.475 0.296	−3.220 0.482	−7.953 0.384	−2.953 0.484
DCC-ARE					-	−1.860 0.426	6.034 0.598	−10.211 0.384	5.078 0.602	0.371 0.526	5.345 0.606
DCC-TVR						-	7.924 0.604	−8.320 0.408	6.932 0.606	2.206 0.540	7.201 0.618
DECO							-	−16.086 0.002	−1.207 0.532	−6.151 0.060	−0.930 0.564
DEC-AVE								-	14.799 0.958	9.855 0.968	15.077 0.956
DEC-TVV									-	−5.009 0.244	0.275 0.752
DEC-ARE										-	5.173 0.758
DEC-TVR											-

Table B.29: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of European indices, high volatility sub-period of Jul 2007 to Dec 2011 (2360:3265).

European Indices: Relative economic value, second low volatility sub-period

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	−9.789 0.316	6.299 0.564	−9.798 0.316	−6.775 0.386	−10.157 0.314	−62.606 0.220	−51.177 0.260	−57.042 0.246	−60.560 0.222	−57.042 0.246
DCC		-	16.188 1.000	−0.007 0.488	2.953 0.932	−0.346 0.424	−54.060 0.262	−42.586 0.290	−48.530 0.272	−52.039 0.262	−48.530 0.272
DCC-AVE			-	−16.295 0.000	−13.351 0.006	−16.636 0.000	−70.116 0.238	−58.584 0.262	−64.607 0.250	−68.110 0.238	−64.607 0.250
DCC-TVV				-	2.959 0.924	−0.339 0.430	−54.057 0.262	−42.582 0.292	−48.527 0.272	−52.035 0.262	−48.527 0.272
DCC-ARE					-	−3.310 0.198	−57.054 0.260	−45.590 0.288	−51.520 0.268	−55.024 0.262	−51.521 0.268
DCC-TVR						-	−53.730 0.262	−42.256 0.294	−48.200 0.270	−51.709 0.264	−48.201 0.270
DECO							-	11.803 0.990	5.496 0.990	1.892 0.790	5.495 0.990
DEC-AVE								-	−6.396 0.100	−9.996 0.150	−6.397 0.100
DEC-TVV									-	−3.656 0.398	−0.000 0.092
DEC-ARE										-	3.589 0.590
DEC-TVR											-

Table B.30: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of European indices, second low volatility sub-period of Dec 2011 to Dec 2014 (3266:3917).

European Indices: Relative economic value, sub-period ending 2012

	CCC	DCC	DCC-AVE	DCC-TVV	DCC-ARE	DCC-TVR	DECO	DEC-AVE	DEC-TVV	DEC-ARE	DEC-TVR
CCC	-	37.208 0.952	49.169 0.978	21.567 0.852	31.521 0.936	30.848 0.906	−71.089 0.006	−39.728 0.072	−63.868 0.010	−77.725 0.006	−63.549 0.010
DCC		-	12.038 0.988	−15.559 0.002	−5.908 0.014	−6.346 0.324	−109.168 0.002	−77.801 0.016	−101.927 0.006	−116.144 0.002	−101.615 0.006
DCC-AVE			-	−27.650 0.000	−18.022 0.014	−18.445 0.144	−121.190 0.002	−89.812 0.006	−113.959 0.002	−128.184 0.000	−113.647 0.002
DCC-TVV				-	9.551 0.964	9.130 0.878	−93.728 0.006	−62.372 0.024	−86.492 0.010	−100.714 0.004	−86.180 0.010
DCC-ARE					-	−0.465 0.510	−103.369 0.004	−72.006 0.018	−96.121 0.006	−110.318 0.004	−95.808 0.006
DCC-TVR						-	−107.320 0.012	−75.955 0.042	−100.079 0.016	−114.296 0.012	−99.767 0.016
DECO							-	31.412 1.000	7.129 0.990	−7.477 0.004	7.432 0.992
DEC-AVE								-	−24.399 0.000	−39.040 0.000	−24.096 0.000
DEC-TVV									-	−14.696 0.000	0.302 0.898
DEC-ARE										-	14.833 1.000
DEC-TVR											-

Table B.31: Estimated relative economic value gained from moving from the forecast in the row heading to that in the column heading, for $\mu = 6\%$, $\lambda = 2$. Each entry reports the average values of δ across 500 bootstraps and the proportion of bootstraps where δ is positive. Portfolio of European indices, period spans 4 June 1996 to 31 December 2012.

Details of European Indices, including summary statistics

Country	Min	Max	$\bar{x}$	s	Skewness	Kurtosis
Belgium	-0.110	0.097	0.000	0.014	-0.194	10.158
Denmark	-0.138	0.118	0.000	0.014	-0.507	12.076
Estonia	-0.156	0.146	0.000	0.017	-0.244	19.637
France	-0.095	0.105	0.000	0.016	-0.049	6.985
Germany	-0.098	0.116	0.000	0.017	-0.130	6.772
Greece	-0.139	0.154	0.000	0.021	-0.143	8.043
Hungary	-0.155	0.241	0.000	0.019	-0.009	16.068
Norway	-0.135	0.094	0.000	0.015	-0.820	11.608
Poland	-0.102	0.122	0.000	0.016	-0.119	7.803
Spain	-0.116	0.135	0.000	0.016	-0.128	7.552
Sweden	-0.120	0.117	0.000	0.015	-0.170	8.161
Switzerland	-0.077	0.075	0.000	0.013	-0.247	7.375
Turkey	-0.203	0.266	0.000	0.028	-0.074	10.901
UK	-0.087	0.108	0.000	0.013	-0.012	8.465
VIX	9.890	80.860	21.313	8.480	1.864	9.055

Table B.32: List of 14 European indices and VIX summary statistics, entire period spans 4 June 1996 to 31 December 2014.

U.S. Equities: Portfolios

$N = 5$									
AA	ABT	ADM	AEP	AET					
$N = 10$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
$N = 25$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC					
$N = 50$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC	CSX	D	DD	DOV	DOW
DOW	DTE	DUK	ED	EMR	ETN	ETR	EXC	GCI	GD
GE	GIS	GLW	GPC	GPS	GWW	HAL	HNZ	HON	HRB
$N = 100$									
AA	ABT	ADM	AEP	AET	AIG	AMD	APD	ATI	AVP
AXP	BA	BAX	BCR	BDX	BLL	CAG	CAT	CB	CI
CL	CLX	COP	CPB	CSC	CSX	D	DD	DOV	DOW
DTE	DUK	ED	EMR	ETN	ETR	EXC	GCI	GD	GE
GIS	GLW	GPC	GPS	GWW	HAL	HNZ	HON	HRB	HSY
IFF	IP	ITT	ITW	JCI	JCP	JPM	K	KMB	KR
LLY	LMT	LNC	MAS	MCD	MDP	MDT	MHP	MRK	NEM
NOC	NSC	PBI	PCG	PEG	PEP	PFE	PG	PH	PHM
PPG	R	RDC	RTN	SLB	SNA	SVU	SWK	T	THC
TJX	TXT	UTX	VFC	WBA	WFC	WHR	WMB	WMT	WY

Table B.33: List of stocks included in each portfolio for U.S. equities dataset.

Details of U.S. Equities, including summary statistics

Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
AA	Alcoa Inc.	Materials	-0.175	0.209	0.000	0.027	-0.091	9.930
ABT	Abbott Laboratories	Health Care	-0.176	0.218	0.000	0.017	0.242	15.843
ADM	Archer-Daniels-Midland Co.	Consumer Staples	-0.184	0.160	0.000	0.021	-0.161	11.660
AEP	American Electric Power	Utilities	-0.161	0.181	0.000	0.016	0.261	17.294
AET	Aetna Inc.	Health Care	-0.227	0.254	0.000	0.024	-0.485	15.440
AIG	American Intl Group	Financials	-0.460	0.460	0.000	0.042	-0.505	46.914
AMD	Advanced Micro Devices	Information Technology	-0.392	0.232	0.000	0.042	-0.421	10.247
APD	Air Products & Chemicals Inc.	Materials	-0.131	0.137	0.000	0.019	-0.097	7.653
ATI	Allegheny Technologies Inc.	Materials	-0.213	0.229	0.000	0.033	-0.113	6.856
AVP	Avon Products Inc.	Consumer Staples	-0.324	0.176	0.000	0.023	-0.744	20.052
AXP	American Express Co.	Financials	-0.194	0.188	0.000	0.025	0.008	10.329
BA	The Boeing Co.	Industrials	-0.194	0.144	0.000	0.021	-0.362	9.427
BAX	Baxter International Inc.	Health Care	-0.186	0.104	0.000	0.018	-0.950	13.018
BCR	CR Bard Inc.	Health Care	-0.124	0.198	0.000	0.016	0.337	13.186
BDX	Becton Dickinson & Co.	Health Care	-0.252	0.158	0.000	0.018	-0.584	18.483
BLL	Ball Corp.	Materials	-0.108	0.110	0.001	0.019	0.234	7.493
CAG	Conagra Foods Inc.	Consumer Staples	-0.217	0.104	0.000	0.016	-0.739	17.182
CAT	Caterpillar Inc.	Industrials	-0.157	0.137	0.001	0.022	-0.097	6.710
CB	Chubb Corp.	Financials	-0.134	0.155	0.000	0.019	0.398	10.513
CI	Cigna Corp.	Health Care	-0.247	0.211	0.000	0.024	-0.654	17.627
CL	Colgate-Palmolive Co.	Consumer Staples	-0.173	0.182	0.000	0.016	-0.005	14.394
CLX	Clorox Company	Consumer Staples	-0.176	0.124	0.000	0.017	-0.422	12.934
COP	Conocophillips	Energy	-0.149	0.154	0.000	0.019	-0.298	8.755
CPB	Campbell Soup Co.	Consumer Staples	-0.144	0.183	0.000	0.016	0.368	14.159
CSC	Computer Sciences Corp.	Information Technology	-0.254	0.170	0.000	0.025	-0.491	12.550
CSX	CSX Corp.	Industrials	-0.176	0.141	0.000	0.022	-0.105	7.224
D	Dominion Resources Inc./VA	Utilities	-0.137	0.100	0.000	0.014	-0.584	13.132
DD	DU Pont (E.I.) de Nemours	Materials	-0.120	0.109	0.000	0.020	-0.155	6.953
DOV	Dover Corp.	Industrials	-0.178	0.146	0.000	0.020	-0.113	7.784
DOW	The Dow Chemical Co.	Materials	-0.211	0.169	0.000	0.023	-0.239	9.509
DTE	DTE Energy Co.	Utilities	-0.111	0.122	0.000	0.014	0.045	10.559
DUK	Duke Energy Corp.	Utilities	-0.161	0.150	0.000	0.016	-0.184	13.673
ED	Consolidated Edison Inc.	Utilities	-0.070	0.090	0.000	0.012	0.149	7.681

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Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
EMR	Emerson Electric Co.	Industrials	-0.163	0.143	0.000	0.019	-0.034	8.864
ETN	Eaton Corp. PLC	Industrials	-0.165	0.174	0.000	0.019	-0.090	9.026
ETR	Entergy Corp.	Utilities	-0.196	0.133	0.000	0.016	-0.396	15.074
EXC	Exelon Corp.	Utilities	-0.125	0.159	0.000	0.017	-0.019	10.990
GCI	Gannett Co. Inc.	Consumer Discretionary	-0.274	0.332	0.000	0.025	0.119	23.915
GD	General Dynamics Corp.	Industrials	-0.132	0.111	0.000	0.017	-0.221	7.138
GE	General Electric Co.	Industrials	-0.137	0.180	0.000	0.020	0.020	9.915
GIS	General Mills Inc.	Consumer Staples	-0.119	0.090	0.000	0.012	-0.416	11.185
GLW	Corning Inc.	Information Technology	-0.434	0.196	0.000	0.034	-0.618	14.420
GPC	Genuine Parts Co.	Consumer Discretionary	-0.095	0.096	0.000	0.014	0.171	7.235
GPS	The Gap Inc.	Consumer Discretionary	-0.236	0.241	0.000	0.027	-0.308	11.317
GWW	WW Grainger Inc.	Industrials	-0.147	0.159	0.000	0.018	0.154	8.994
HAL	Halliburton Co.	Energy	-0.303	0.213	0.000	0.030	-0.276	10.140
HNZ	HJ Heinz Co.	Consumer Staples	-0.088	0.103	0.000	0.014	0.134	8.186
HON	Honeywell International Inc.	Industrials	-0.196	0.254	0.000	0.022	-0.254	13.913
HRB	H&R Block Inc.	Consumer Discretionary	-0.197	0.171	0.000	0.022	-0.436	10.825
HSY	The Hershey Co.	Consumer Staples	-0.128	0.225	0.000	0.016	0.734	18.740
IFF	Intl Flavors & Fragrances	Materials	-0.174	0.149	0.000	0.017	-0.316	12.160
IP	International Paper Co.	Materials	-0.205	0.198	0.000	0.024	0.057	10.167
ITT	ITT Corp.	Industrials	-0.117	0.218	0.000	0.019	0.547	12.318
ITW	Illinois Tool Works	Industrials	-0.101	0.123	0.000	0.018	0.098	6.708
JCI	Johnson Controls Inc.	Consumer Discretionary	-0.133	0.128	0.000	0.021	-0.021	7.627
JCP	J.C. Penney Co. Inc.	Consumer Discretionary	-0.221	0.172	0.000	0.027	0.157	7.761
JPM	JP Morgan Chase & Co.	Financials	-0.232	0.224	0.000	0.027	0.209	13.331
K	Kellogg Co.	Consumer Staples	-0.101	0.103	0.000	0.015	0.118	8.745
KMB	Kimberly-Clark Corp.	Consumer Staples	-0.120	0.101	0.000	0.015	-0.254	10.342
KR	Kroger Co.	Consumer Staples	-0.295	0.097	0.000	0.020	-1.004	18.572
LLY	Eli Lilly & Co.	Health Care	-0.193	0.163	0.000	0.019	-0.157	10.535
LMT	Lockheed Martin Corp.	Industrials	-0.148	0.137	0.000	0.018	-0.190	9.915
LNC	Lincoln National Corp.	Financials	-0.509	0.362	0.000	0.034	-1.196	46.638
MAS	Masco Corp.	Industrials	-0.174	0.168	0.000	0.025	-0.161	8.362
MCD	McDonald's Corp.	Consumer Discretionary	-0.137	0.103	0.000	0.017	-0.091	8.077
MDP	Meredith Corp.	Consumer Discretionary	-0.148	0.152	0.000	0.020	0.081	10.073

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Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
MDT	Medtronic Inc.	Health Care	-0.142	0.112	0.000	0.019	-0.192	8.364
MHP	McGraw-Hill Companies Inc.	Financials	-0.142	0.214	0.000	0.020	0.276	12.259
MRK	Merck & Co. Inc.	Health Care	-0.194	0.231	0.000	0.019	-0.088	15.910
NEM	Newmont Mining Corp.	Materials	-0.187	0.225	0.000	0.028	0.448	8.378
NOC	Northrop Grumman Corp.	Industrials	-0.158	0.213	0.000	0.017	0.154	15.028
NSC	Norfolk Southern Corp.	Industrials	-0.138	0.143	0.000	0.022	-0.016	6.371
PBI	Pitney Bowes Inc.	Industrials	-0.190	0.208	0.000	0.019	-0.538	16.064
PCG	P G & E Corp.	Utilities	-0.224	0.269	0.000	0.021	0.391	32.827
PEG	Public Service Enterprise Gp	Utilities	-0.110	0.158	0.000	0.016	0.062	11.429
PEP	Pepsico Inc.	Consumer Staples	-0.127	0.156	0.000	0.016	0.241	12.198
PFE	Pfizer Inc.	Health Care	-0.118	0.097	0.000	0.018	-0.214	6.648
PG	The Proctor & Gamble Co.	Consumer Staples	-0.159	0.097	0.000	0.015	-0.519	11.513
PH	Parker Hannifin Corp.	Industrials	-0.126	0.128	0.000	0.022	-0.077	6.595
PHM	Pultegroup Inc.	Consumer Discretionary	-0.204	0.207	0.000	0.031	0.126	6.852
PPG	PPG Industries Inc.	Materials	-0.122	0.138	0.000	0.019	0.124	7.578
R	Ryder System Inc.	Industrials	-0.198	0.125	0.000	0.023	-0.388	8.682
RDC	Rowan Companies PLC	Energy	-0.217	0.224	0.000	0.032	-0.120	6.169
RTN	Raytheon Company	Industrials	-0.215	0.237	0.000	0.020	-0.179	19.148
SLB	Schlumberger Ltd	Energy	-0.203	0.139	0.000	0.025	-0.275	7.350
SNA	Snap-On Inc.	Industrials	-0.175	0.141	0.000	0.019	0.027	10.739
SVU	Supervalu Inc.	Consumer Staples	-0.274	0.383	0.000	0.026	-0.447	27.695
SWK	Stanley Black & Decker Inc.	Industrials	-0.146	0.140	0.000	0.021	0.192	7.441
T	AT & T Inc.	Telecommunication Services	-0.135	0.151	0.000	0.018	0.087	7.701
THC	Tenet Healthcare Corp.	Health Care	-0.335	0.439	0.000	0.032	-0.086	29.592
TJX	TJX Companies Inc.	Consumer Discretionary	-0.174	0.163	0.001	0.023	0.059	7.940
TXT	Textron Inc.	Industrials	-0.381	0.398	0.000	0.027	-0.751	33.175
UTX	United Technologies Corp.	Industrials	-0.187	0.128	0.001	0.018	-0.209	9.149
VFC	VF Corp.	Consumer Discretionary	-0.146	0.136	0.001	0.019	0.186	7.942
WBA	Walgreen Co.	Consumer Staples	-0.162	0.155	0.000	0.019	-0.026	8.699
WFC	Wells Fargo & Co.	Financials	-0.272	0.283	0.000	0.026	0.769	25.109
WHR	Whirlpool Corp.	Consumer Discretionary	-0.155	0.176	0.000	0.024	0.150	7.664
WMB	Williams Cos Inc.	Energy	-0.373	0.373	0.000	0.034	-0.802	28.017
WMT	Wal-Mart Stores Inc.	Consumer Staples	-0.107	0.109	0.000	0.018	0.141	7.136

continued on next page

Ticker	Company Name	Sector	Min.	Max.	$\bar{x}$	s	Skewness	Kurtosis
WY	Weyerhaeuser Co.	Financials	-0.188	0.131	0.000	0.022	-0.145	7.310
VIX	S&P 500 Volatility Index		9.890	80.860	22.036	8.505	1.931	9.586

Table B.34: Details of the 100 stocks included in the full dataset and VIX summary statistics, entire period spans 3 January 1996 to 31 December 2012.

Appendix C

Chapter 5 Supplement

Univariate parameter estimates

Stock	μ	φ	α	β	ϕ
ANZ	5.1060	0.5347	0.0715	0.8695	0.0168
	(1.4946)	(0.0426)	(0.0060)	(0.0111)	(0.0049)
BHP	$-$ 7.5116	-0.1002	0.0487	0.9396	0.0080
	(3.3725)	(0.0311)	(0.0049)	(0.0064)	(0.0031)
NAB	-15.0973	0.6318	0.0746	0.8626	0.0131
	(2.4185)	(0.0373)	(0.0095)	(0.0129)	(0.0195)
RIO	4.3955	0.4815	0.0801	0.8675	0.0113
	(1.3787)	(0.0522)	(0.0069)	(0.0128)	(0.0029)
WOW	4.3654	0.3862	0.0693	0.8730	0.0070
	(3.9264)	(0.0800)	(0.0126)	(0.0236)	(0.0106)

Table C.1: Univariate intraday volatility model parameter estimates and robust standard errors for each stock, see Section 5.2.1 of Chapter 5. Entire period spans 4 January 2011 to 29 December 2012.

Details of dataset, including summary statistics

Stock	Min	Max	$\bar{x}$	s	Skewness	Kurtosis
ANZ	-0.0151	0.0144	0.0000	0.0012	0.1414	9.3257
BHP	-0.0077	0.0078	0.0000	0.0009	0.0000	6.7023
NAB	-0.0120	0.0123	0.0000	0.0012	0.0753	8.5173
RIO	-0.0101	0.0115	0.0000	0.0011	0.0412	8.6888
WOW	-0.0127	0.0110	0.0000	0.0009	-0.0250	7.8445

Table C.2: List of 5 Australian stocks and summary statistics, period spans 4 January 2011 to 29 December 2012.

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